Entering the mathematical register through evolution of the material milieu for classification of polygons

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This paper is based on classroom observations from Norway with 7-8 years old children working on geometrical shapes. The intention is that the children shall classify different polygons according to their number of edges. The observations are part of a teaching sequence that is designed using principles from Brousseau's Theory of Didactical Situations (TDS). From the teaching sequence we identify certain challenges in the children's development of scientific terms and the observations allow us to conclude that these challenges to some extent are connected to specific semantic features of the Norwegian language. The use of TDS is instrumental in revealing the challenges that occurred and explaining what was changed to overcome them.

Keywords: Register, language of nearness and distance, polygons, adidactical situation, milieu.

Introduction

This paper reports on a teaching sequence within the project Language Use and Development in the Mathematics Classroom (LaUDiM)—an intervention study carried out in collaboration between researchers at the Norwegian University of Science and Technology and two local primary schools in the period 2014-2018. The main objective of the project is to study pupils' development and use of mathematical language in order to gain knowledge that will help the teachers to develop their teaching—aimed at pupils' increased proficiency in expressing mathematical ideas, mathematical reasoning, arguing and justification. Teaching sequences are designed in collaboration between researchers and teachers, where the design is guided by principles from the theory of didactical situations (Brousseau, 1997).

In this paper we study a teaching sequence at one of the project schools including pupils from Grade 2 (7-8 years old) and their teacher. The main aim of the teaching sequence—consisting of three sessions—is that the pupils shall develop their language use about polygons, with the specific aim that they shall be able to classify polygons based on seeing visual images and that they can discern different parts of a polygon (vertices and edges). The classroom observations (recorded on video) give insights into pupils' evoked concept images (Tall & Vinner, 1981) of vertex and edge, and how discrepancy between these and the scientific definitions of them constrains the teacher's goal of the first session. Further, we show how this is resolved by the teacher in the subsequent sessions.

Theoretical framework

According to Halliday a register is "a configuration of meanings that are typically associated with a particular configuration of field, mode and tenor" (1985, pp. 38-39). Halliday compares *register* to *dialect* and states that "dialects are saying the same thing in different ways, whereas registers are saying different things" (Halliday, p. 41). So changing between registers can mean that the same word gets a different meaning. The mathematical register is characterised by the property that words have very precise meanings and sometimes the same word may be used in the mathematical register and in the register of everyday language but with different meaning. This feature is language specific in the sense that for a given word it can be present in one language but when this word is translated to another language it may lead to one word in the everyday register and another word in the mathematical register.

Koch and Oesterreicher (1985) make a distinction between language being *conceptually oral* or *conceptually written.* They refer to the first category as *a language of nearness*, where the interlocutors are in direct contact and can comment on each other's utterances and directly refer to the given situation, for instance by using gestures. The second category they refer to as *a language of distance*, where the sender and the receiver are not necessarily in contact and the language therefore has to be more precise. The mathematical language is in its nature conceptually written because of the way it strives for precision and unambiguity. However, much of the communication in the mathematics classroom has many of the features characterising a conceptually oral language, a language of nearness. In particular, when working with young children the communication is characterised by dialogue, face-to-face interaction and a desire to avoid complexity, features characterising a language of nearness. However, one of the aims of schooling is to develop the mathematical language into a language characterised by greater precision, compactness, density of information, features characterising a language of distance (Koch & Oesterreicher, 1985, p. 23).

The theory of didactical situations in mathematics, TDS (Brousseau, 1997) is a scientific approach to the problems related to teaching and learning of mathematics, where the particularity of the knowledge taught plays a significant role. Its methodology—for a targeted piece of mathematical knowledge—is based on creating a *situation* with a problem to be solved, where the knowledge aimed at is the optimal solution to the given problem. In the following, based on Brousseau (1997), we explain some concepts of TDS that are relevant for our analysis.

An *adidactical situation* is a situation in which the student takes a mathematical problem as his own and tries to solve it without the teacher's guidance and without didactical reasoning (i.e., not trying to interpret the teacher's intention with it). The *milieu* models the elements of the material and intellectual reality on which the students act when solving a problem—these elements are conditions for the students' actions and reasoning. The milieu may comprise: the problem to be solved; material or symbolic tools provided (artefacts, informative texts, data, etc.); students' prior knowledge; other students; and, arrangement of the classroom and rules for operating in the situation (determinative of who is supposed to interact with whom). The milieu of an adidactical situation is called an *adidactical milieu*. An appropriate adidactical milieu provides feedback to the students, whether their responses are adequate with respect to the knowledge at stake.

After *devolution*, a phase where the teacher has (temporarily) transferred responsibility for solving the problem to the students, four situations (or phases) follow: Situations of action, formulation, and validation are (intentionally) adidactical situations, whereas the situation of institutionalisation is a didactical phase. The situation of *action* is where the students engage with the given problem on the basis of its inner logic, without the teacher's intervention. The students construct a representation of the situation that serves as a "model" that guides them in their decisions. This model is an example of relationships between certain objects or rules that they have perceived as relevant in the situation. The situation of *formulation* is where the students' formulations are useful in order to act indirectly on the (material) milieu—that is, to formulate a strategy enabling somebody else to operate on the milieu. In this situation the teacher's role is to make different formulations "visible" in the classroom. The situation of *validation* is where the students attempt to explain some phenomenon or verify a conjecture. In this situation the teacher's role is to act as a chair of a scientific debate and (ideally) intervene only to structure the debate and try to make the students use more precise mathematical notions. The situation of *institutionalisation* is where the teacher connects the knowledge built by the students—through adidactical interaction with the milieu—to the scholarly and decontextualised forms of knowledge aimed at by the institution.

Methodical approach

Each teaching sequence in the project starts with a planning session where teachers and researchers work together to plan the activities for two classroom sessions, and in particular set the learning goals for the classroom sessions. Activities and actions are planned according to the phases of TDS, devolution, action, formulation, validation, and institutionalisation (Brousseau, 1997). Some days later, the first classroom session takes place, immediately followed by a reflection session, where experiences from the first classroom session are discussed and adjustments are made for the second session, taking place yet a couple of days later. Researchers are also present in the classroom. Observations from all sessions (planning, classroom implementation and reflection) are recorded on video, and additional audio recording is used to secure the quality of the sound. In the classroom sessions selected pupils working in groups (2-3) are video recorded, as is the teacher in whole-class sessions. Tasks given to the pupils in the observed sessions and written material produced by the pupils are also data sources. After completing a cycle of planning, reflection and classroom sessions, teachers and researchers meet to watch parts of the video recordings from the classroom. This represents the first step in analysing data, where interesting sequences from the classroom are identified. In the planning session and the video session, teachers from both schools are present.

This paper is based on a teaching sequence on geometrical shapes, consisting of three classroom sessions.[1] Data from the teaching sequence form the basis for answering the following research question: *What conditions enable or hinder pupils' opportunities to categorise polygons according to their number of edges?*

The utterances reproduced are excerpts from a transcript of the video recorded whole-class discussion in the second session. The camera faces the teacher at the board and it captures the dialogue between the teacher and the 14 pupils who are sitting in a semi-circle close to the board. Parts of the dialogue between teacher and pupils have been transcribed and translated from

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[1] The analysed teaching sequence consists of three sessions (instead of two which is common in the project). The third session involves pupils' interaction with a milieu designed so as to give feedback in the devolved adidactical situation.

Norwegian into English. In cases where it is important for the analysis to emphasise the meaning of a particular word in Norwegian, the Norwegian word is included in square brackets in the transcript.

Our analysis is based on ethnomethodological conversation analysis, focusing on the thematic development of an interaction rather than on its structural development (Holstein & Gubrium, 2005). This gives the possibility to analyse the relationship between language and the figures with their components while teachers and students negotiate mathematical meaning (Fetzer & Tiedemann, 2015).

The teaching sequence analysed here is chosen because: (1) it illustrates how the phenomenon of words having different meanings in the mathematical and everyday registers constrains pupils' conceptual development; and (2) it illustrates how an evolution of the milieu gives a rationale for *using* the target knowledge.

Analysis of the teaching sequence

First session—classification

In the first session of the teaching sequence, pupils work in pairs on sheets of paper showing 12 shapes, as presented in Figure 1 (one pupil has blue, the other has red figures). The task they get is that each pupil shall (individually) sort the figures into groups (cutting the individual figures from the sheet) and give a name to each group (ACTION). Then they are supposed to compare (in the pairs) how they have sorted the figures and agree on a way to sort them and also agree on a name for each group (FORMULATION). The final result from each group is a sheet of paper on which the pupils have glued on figures from the same group and with the text "These are <__> because <____>" and the pupils have filled in the blanks (VALIDATION). After the session the teacher collects the worksheets and she uses them as background for a whole-class discussion in the second session (INSTITUTIONALISATION).

In Norwegian, polygons are named literally after the number of edges, using the standard Norwegian number words, so that a triangle is called a "three edge" (trekant), a quadrilateral is called a "four edge" (firkant), and similarly for the others. An accepted name for the generic concept *polygon* is 'mangekant' which literally means "many edge". Learning names of polygons, and understanding the reason for the names, is therefore not considered to be a challenge for Norwegian students.

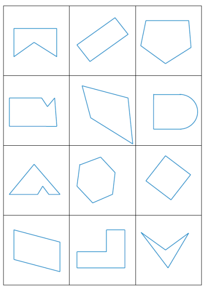


Figure 1: Shapes to be classified

This is in contrast to the situation in English where it is not obvious from the everyday language that for instance a pentagon is a shape with five edges. The teacher has seen from the collected worksheets that all groups have given names to the shapes based on the number of edges and they have written for instance "these are 'five edges' because they have five edges". However, from the discussion in pairs she has observed that even if all the pupils talk about *edges* (kanter), the way they point at the figures indicates that some counts the edges but others count the vertices. The Norwegian language has no precise scientific word for *vertex,* the word which is used is 'hjørne', which (also) means *corner*.

Second session—the meanings of edge and corner

In the institutionalisation phase the teacher asks pupils to come to the board and explain their reasoning. She has observed Oliver and Amelia counting the vertices and Thomas, Daniel and Sophie counting the edges. Among the figures is a quadrilateral with three acute angles and one reflex angle (Figure 2), where we have inserted the letter A for reference in the dialogue. Although Oliver and Amelia have grouped this among the quadrilaterals, Oliver expresses some doubt when he is called to the board to explain how he and Amelia have thought.

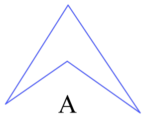


Figure 2: Non-convex quadrilateral

Oliver:	If we had pulled this out a little (pointing to vertex A with the reflex angle) it would have been a "four edge" [firkant].
Teacher:	OK, but still you have grouped this among the "four edges".
Oliver:	One, two three, four (pointing to the vertices).
Teacher:	So what is an edge?
Oliver:	That is the pointed parts [spissene].

When Thomas is called to the board he uses a rectangle as his example and clearly points to the edges, counting "one-two-three-four".

Teacher:	What is the difference between what Oliver did and what Thomas did?
Megan:	Thomas counted the lines [strekene] and Oliver counted the pointed parts [spissene].
Teacher:	So actually we did not quite agree on what an edge really is.

The dialogue above reveals that there are different opinions among the pupils as to what the word 'kant' means. All the pupils claim that they are counting the edges but when asked to explain what they have counted, Oliver points to the vertices and Thomas points to the edges. Megan makes the observation that the two boys have actually counted different parts of the polygon.

The teacher has in many sessions talked about "what mathematicians do" and that they for instance decide and agree on what names to give to mathematical objects. At this stage the teacher says that now we have to agree on something—as the mathematicians do—so that we have a common understanding of what an edge is. The teacher has also observed that some pupils use the word 'hjørne' and she draws their attention to this. One pupil, Jessica, says that they had talked about 'hjørne' but they did not know what it was, so they had written 'kanter'. The teacher asks the pupils to explain what a 'hjørne' is and encourages William to come to the board.

William:	That inside is a corner and those outside are edges.
Teacher:	Can you show us?
William:	This is a corner (points to vertex A with the reflex angle in Figure 2) and that is an edge (points to one of the acute angles).
Teacher:	But what about this (points to the rectangle)?
Oliver:	Edge, edge, edge, edge (points to each of the four vertices).

Then the pupils continue to discuss the inner and the outer angle at a vertex, and that one is a ‘hjørne’ and the other is a ‘kant’. Thomas says that “the corners are inside and the edges are outside” and Chloe agrees that the corner is inside but she refers to the outside as the “pointed parts” (spissene). William is making a distinction between the vertex at the reflex angle of the non-convex quadrilateral (Figure 2), which he refers to as a ‘hjørne’, and the vertices at the acute angles, which he denotes by the word ‘kant’. Oliver, using the rectangle as his reference context, refers to all the vertices by the word ‘kant’.

To get the pupils to agree on *one* name for the same object the teacher brings in a reference context from their everyday life, a mini-pitch. The picture in Figure 3 is shown on the whiteboard. Using this picture as the reference context, the teacher asks questions like “If I say that you should place yourself on the edge of the mini-pitch, where would you be standing?” or “…place yourself at the corner, where would you go?”

Figure 3: The mini-pitch

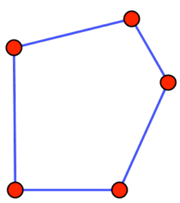
There is still some confusion among the pupils, so the teacher says that she will tell them “what the mathematicians have decided”. She holds up a rectangular sheet of paper (A4) and says:

Teacher: Corner (vertex), that is where two sides meet. When we talk about edge, we can also call this the side-edge [sidekant], and where two edges meet, that is a corner (vertex). There is the corner (points to a vertex of the sheet).

The last part of the session was completed at the mini-pitch in the schoolyard, where they played the game of the teacher telling where to go—by using the concepts of edge and corner—and the pupils went to a place which (supposedly) fulfilled the teacher’s command.

The teacher’s reference to “work like mathematicians” and that this entails giving precise definitions indicates that she intends to introduce her pupils to the mathematical register. However, the communication is hindered by the fact that some of the words have different meanings in the mathematical register and the everyday register. In particular this is the case for the word ‘hjørne’ which can mean vertex (mathematical register) as well as corner (everyday register). In everyday language the corner is a spacious area, somewhere you can stand, but in mathematics it is a point, the intersection between two lines. The word ‘kant’ has the same connotation as English edge, and in everyday language this is used as something that is sharp. This may explain why ‘kant’ is used to denote both the side, or edge, and the vertex when it is approached from the outside.

Third session—a milieu that affords feedback



As a follow up, a game with 12 tiles was developed. On one side of the tile was depicted a polygon where the edges had one colour and the vertices were marked with another colour. On the reverse side was written “<*name of shape*> with <*colour*> edges” or “<*name of shape*> with <*colour*> vertices”. An example is shown in Figure 4. On the back of this was written “Pentagon (femkant) with blue edges”.

Figure 4: Pentagon with blue edges

This game was played in pairs of pupils both having the full set of tiles. One pupil reads the text and the other one is supposed to pick the correct shape. After picking he/she can turn the tile and read the text to see if the correct shape has been picked.

Discussion

The target knowledge of the teaching sequence was that the pupils should develop the scientific language for naming 2D shapes and become aware that these names are based on the number of edges in the shape. To know the difference between edges and corners (vertices) will then also be part of the target knowledge. A condition that hinders pupils' opportunity to categorise polygons according to their number of edges, is the ambiguous use of the concept of edge. Many of the pupils thought that edge ('kant') referred to corner/vertex, and since the number of edges equals the number of vertices it gave meaning to classify polygons according to the number of corners.

The material milieu in Session 1 did not have an adidactical potential for categorisation according to the number of edges, since it was possible to solve the task apparently correct, without the pupils having a common understanding of what is an edge and what is a corner/vertex. There was no feedback from the milieu that could have told them whether they used the desired concept to classify: In action they counted either edges or corners (which gave the same answer); in formulation they compared their categorisations (and if they had a figure that was categorised differently, they used either edges or corners as a basis for categorising jointly and agreeing); in validation, if they reasoned on the basis of different attributes (edges or corners), they concluded that it did not matter which attribute to use.

During Session 1, the teacher realised that the pupils had other connotations of edge and corner/vertex than the scientific ones. In institutionalisation (Session 2), the teacher let the different connotations be displayed, and—with reference to mathematicians—she introduced the scholarly meaning of the concepts, in the mathematical register. Further, she connected them to the pupils' everyday register, through the mini-pitch context.

Based on results from Sessions 1 and 2, the teacher designed a new material milieu (tiles) that has an adidactical potential (see Figure 4). The game will produce a win if the pupil uses the scholarly meaning of edge and corner, and a loss if not. Hence, the pupil will *need* the target knowledge to act on the milieu—a principle at the core of TDS' instructional design. The evolution of the milieu described here is a condition that *enables* the pupils' opportunity to categorise polygons according to their number of edges.

The teacher's desire to introduce precise mathematical terms also points to introducing a language of distance. However, the situation is such that the pupils are able to express themselves clearly using gestures together with oral language, thereby using a language of nearness. However, in the game with the tiles it is necessary to use a language of distance in order to pick the correct tile. Hence, the intended language development is stimulated by the activity's adidactical potential.

It has been observed earlier that Norwegian children focus on the vertices when naming polygons (Rønning, 2004) but the observations made in this paper show that they may use different words depending on whether they approach the vertex from the inside or from the outside. We have also seen that they may use different words within the same shape, as with the non-convex quadrilateral in Figure 2. This shape is also interesting in the sense that it is not really accepted by Oliver as a quadrilateral but it would have been if "we had pulled this [the vertex with the reflex angle] out a little". We interpret this as Oliver's inclination to distinguish between convex and non-convex polygons. In future learning of geometry, the concept of a convex polygon will be introduced and this example indicates that early exposition to non-convex shapes can be important for making pupils familiar with these shapes.

Distinguishing between edges and vertices can also be seen to be important for future learning. For polygons, the number of edges is equal to the number of vertices so to name a polygon one may just as well count the number of vertices instead of the number of edges. However, for polyhedra the number of vertices, edges and faces are not the same, and the naming is based on the number of faces.

The results presented here are relevant for mathematics teachers and teacher educators: They present challenges and affordances related to teaching of properties of polygons—with emphasis on language and characteristics of the milieu with which the pupils interact when solving a problem.

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