Language Use and Development in the Mathematics Classroom
(LaUDiM)

PART 1: Knowledge needs

1. Knowledge needs
Mathematics is a key competence in our society, both as a prerequisite for being an active participant in a democratic society, and as a basis for further development in science and technology. This stance is a basis for the Norwegian curriculum, *Kunnskapsløftet* (LK06) and hence needs to be reflected in teacher education. Furthermore, it is acknowledged that children's acquisition of number sense in early years is a strong predictor of later mathematical success (Aubrey, Dahl, & Godfrey, 2006; Aunio & Niemvirta, 2010).

The two white papers (St. meld. nr. 16, 2006-2007, *...og ingen sto igjen. Tidlig innsats for livslang læring* and St. meld. nr. 31, 2007-2008, *Kvalitet i skolen*) express the need to give priority to early effort for lifelong learning in school. This proposal is based on research showing that learning in early years is crucial, and on documentation showing that many pupils do not appropriate the basic mathematical skills in early years as desired (OECD, 2013). According to a number of national policy documents the connection between pupils' learning outcomes and teachers' teaching and assessment practices is in need of strengthening (*LK06*; St.meld. nr. 31, 2007-2008, *Kvalitet i skolen*; St.meld. nr. 30, 2003-2004, *Kultur for læring*, Meld. St. 20, 2012-2013, *På rett vei*). The proposed project will be carried out as an intervention study where teachers and researchers collaborate on exploring and developing teaching and learning of mathematics. Based upon existing research on teaching and learning of mathematics, we intend to develop further knowledge on how to prepare for and conduct teaching in an inquiry oriented mathematics classroom (Jaworski, 2006) in close collaboration with teachers and student teachers.

There is a substantial knowledge body about the importance of language for learning in general, and for learning mathematics in particular. Despite this knowledge there is evidence to support the claim that much of the activity in school does not comply with this knowledge (Olsen, 2013). There is also evidence to support that knowledge from educational research is not readily transmitted to the practice field, and that there is a need for an increased active engagement in research from both teachers and student teachers (Forskningsrådet, 2011).

It has been articulated on the highest political level in Norway that as a rule all teacher education should be on master's level, and it has also been emphasised that a strong background in the subject to be taught is crucial also in the lower grades (Statsministerens kontor, 2013). This project addresses these aims in the sense that the project is strongly linked to HiST's master programme in mathematics education for grades 1-7. Students on this programme will be actively involved as researchers in the project. Further, it will develop research competence among people already employed as teacher educators, as well as recruiting new people to the field through a PhD scholarship.

PART 2: The Knowledge-building Project

2. Objectives
The main objective of the project which relates to the thematical priorities A and B in the FINNUT programme, is to develop a successful learning culture in early learning of mathematics with special emphasis on language development. This comprises the pupils' proficiency in a broad register of mathematical discourse, as well as the teachers' proficiency in orchestrating the mathematics classroom in ways which contribute to the intended learning for pupils. More particular objectives can be described as:

- Improve pupils' proficiency in expressing mathematical concepts and ideas using a variety of representations
- Improve pupils' proficiency in mathematical reasoning, arguing and justification
- Develop teaching practices that fascilitate pupils' mathematical proficiency

We follow Kilpatrick, Swafford, and Findell (2001) in their description of mathematical proficiency as a composite concept made up of five intertwined strands; conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Successful learning of mathematics comes from working consciously with all these strands. This framework also constitutes the theoretical basis of "regning som grunnleggende ferdighet" in Norway (Utdanningsdirektoratet, 2012). In this project we will focus in particular on the role of language and communication in developing mathematical proficiency.

3. Frontiers of knowledge and technology

The project is based on a socio-cultural view on learning, drawing on the work of Vygotsky (1978, 1987) and his followers. We see different aspects of socio-cultural theory to be relevant for our project: learning as *mediated action* (Wertsch, 1991); learning as participation in *communities of practice* (Lave & Wenger, 1991); learning as an *object oriented, collective and culturally mediated activity* (Engeström, 1999; Leont'ev, 1979); and, mathematics teaching development as building *communities of inquiry* (Jaworski, 2008).

In early learning special attention has to be paid to building bridges between the children's everyday language and experiences and the language offered in the school context. As Vygotsky (1934/1987) we see content learning as associated with the interplay between *spontaneous* and *scientific concepts*, and a given word may assume a different function in the two contexts. In the everyday context the word functions as a means of communication but in the scientific context the word itself becomes the object of communicative activity. We build on the practice of *instructional conversation,* developed by Goldenberg and colleagues (Goldenberg, 1991), as a medium for bridging the two kinds of concepts.

Our conceptualisation of teaching is informed by Brousseau's (1997) *theory of didactical situations in mathematics* (hereafter referred to as TDS). Central concepts here are the notions of an *adidactical situation, milieu, didactical contract,* and *institutionalisation*. We will draw on TDS both in the design of interventions for the classroom, and in our analysis of the data. In TDS the teacher is seen to have two roles: one is to devolve to the pupils an adidactical situation with a *milieu* that gives feedback which can enable the pupils' appropriation of the knowledge aimed at (*devolution*); the other is to decontextualise the situated knowledge developed by the pupils, and transform it into cultural knowledge that the pupils can reuse in other situations (*institutionalisation*). Further, we will draw on conceptualisations of conditions for pupils' engagement in the classroom (Måsøval, 2011), and the impact of task design in mathematics (e.g. Mason & Johnston-Wilder, 2006; Måsøval, 2013).

With LK06, reading, writing and mathematics (*regning*), along with oral skills and digital competence, gained status as basic skills in all school subjects. This brought along a new research interest for such key competencies. Still, a recent evaluation of the curriculum states that the basic skills only to a small extent are implemented in the schools' work (Møller et al., 2010). In the proposed project, we focus on early learning of mathematics, where an important objective is to aquire what we will denote as *number sense* (Anghileri, 2000). This incorporates *foundational number sense*, the number-related understandings children develop during the first years of formal instruction, as well as *applied number sense*, which also incorporates the number-related understanding necessary for people to function effectively in society (Andrews & Sayers, 2014). Foundational number sense has been shown to be a more robust predictor of later mathematical success than almost any other factor (Aunio &

Niemivirta, 2010; Mazzocco & Thompson, 2005). In particular, mathematical reasoning, even more than knowledge of arithmetic, is important for children's later achievement in mathematics (Nunes, Bryant, Sylva, & Barros, 2009). It has also been shown (Van Luit & Schopman, 2000) that appropriate intervention can be effective, but that without such intervention, children starting school with limited number sense will remain low achievers throughout their schooling (Aubrey et al., 2006). Obviously, the development of number sense starts long before the children enter school. Hence, the activities in preschool are very important. This is, however, beyond the scope of our project, but links to preschool based research projects will be made (e.g. Helenius & Sterner, 2014).

Research on mathematics in Norwegian classrooms shows that this subject often is taught in traditional ways, focusing on routine skills, memorisation of isolated facts and algorithms, relying strongly on textbooks (e.g. Alseth et al., 2003). Research also indicates that there is little time for dialogues between teachers and pupils, or between pupils (Skorpen, 2006), and that comments from the pupils about mathematics are often not taken up by the teacher (Bjørkås & Bulien, 2010). Most of these studies have focused on organisational aspects of the mathematics classroom. There is thus a need to conduct research on how young pupils learn mathematics, especially through language use and discussions, and how they understand and use mathematical concepts and symbols in their learning. The proposed project builds on previous research done within PraksisFoU/PRAKUT on early learning of mathematics (e.g. Rønning, 2011, 2012). Also within PraksisFoU/PRAKUT there has been done work with preschool children to study mathematical activities in the kindergarten (Carlsen, Erfjord, & Hundeland, 2010). This work is still ongoing at the University of Agder, and the proposed project links to this work through the reference group and network meetings.

4. Research tasks and scientific methods

The proposed project is designed as a four year long intervention-based classroom study in a hermeneutic qualitative research paradigm (Gadamer, 2010; Holter & Kalleberg, 1996). Within this paradigm the researcher recognises possible impacts of subjectivity and personal interactions (Delamont, 1992; Peshkin, 1988). Researchers and teachers enter the process without ready-made solutions but engage in a *co-learning agreement*, where both parts are engaged in actions and reflections (Wagner, 1997). Together researchers and participants change and develop practices as well as co-construct knowledge and understanding.

There is a long tradition of studying small children's development of mathematical concepts within a constructivist/cognitivist framework (Clements & Sarama, 2007). More recently several studies have been carried out focusing on the joint meaning making of the mathematical activity that both teachers and pupils take part in (e.g. Nührenbörger & Steinbring, 2009). In the proposed project use and development of language plays a fundamental role. In mathematics, use of signs and symbols is crucial. As a theoretical framework to study conceptual development through language we will use semiotic theory. Learning is seen as meaning making through mediation between the sign/symbol and the object/reference context. This process is captured by Steinbring (2005) in the *epistemological triangle*, which is inspired by Ogden and Richards (1923/1948) emphasising that the relation between a symbol and its referent is indirect and conventional.

The interventions will be carried out in two primary schools, by planning lessons and designing activities in order to develop a greater understanding of how pupils in Year 1-4 develop mathematical proficiency, and how this connects to teaching and assessment. At each school, teaching and assessment practices will be planned in concert between teachers and researchers, so that each plan may fit its local learning ecology. There are two main objectives of the project regarding teaching and learning (A and B), each with a number of research questions. For analytical purposes we have made a distinction between focusing on the

teachers and on the pupils. In the classroom this is interrelated. At one of the cooperating schools there is a third objective focusing on teacher education (C). In addition we aim to develop knowledge on how video can be used as a tool for fostering user involvement in classroom research (D).

A. *Objectives – focusing on the pupils:* To study pupils' development of mathematical proficiency, with particular focus on the role of language, supported through dialogue and formative assessment.
   Research questions
   - What relationships can be discerned between the *milieu* and the outcome of pupils' engagement with mathematics in the classroom?
   - How do pupils engage in meaning making using different semiotic resources?
   - How can development of procedural and conceptual fluency/knowledge be mutually supportive?
   - How do pupils share and develop knowledge in peer groups?
   - What are characteristic features of pupils' language use at different stages in the research project?

B. *Objectives – focusing on the teachers:* To develop teachers' ability to build and sustain an inquiry oriented mathematics classroom, and explore and develop teachers' assessment of pupils' mathematical proficiency development in Year 1-4.
   Research questions
   - What challenges do teachers face when trying to create a *milieu* where pupils' articulation of mathematics is an important feature?
   - How do teachers interpret and understand pupils' meaning making, and how do they use this insight?
   - What relationships can be discerned between communication patterns in the classroom and pupils' development of mathematical proficiency?
   - What changes can be identified during the project period in teachers' classroom interaction with pupils?
   - What are the teachers' accounts of their own development and their pupils' development of mathematical proficiency?

C. *Objectives – focusing on teacher education:* To explore how the outcome of A and B above can influence and support student teachers' learning through field practice.
   Research questions
   - What are the indicators of student teachers' development of competence as teachers of mathematics?
   - How do student teachers explain their professional development as related to their field practice?

D. *Objectives – focusing on the use of video*
   Research question
   - How can analysis of video sequences of teaching and learning dialogues support teachers' and researchers' involvement in classroom research as a joint enterprise?

In order to address the project's research questions we have chosen a video-based design which promises to be an important tool in intervention studies (Munthe, 2006). Video-design encompasses the complexity and diversity of voices, perspectives and issues at play during teaching and learning in classrooms, and makes it possible to freeze, capture and recapture in detail situations in teaching and learning processes (Klette, 2009). Moreover, in studies of language and communication, details and correct accounts are of great importance, both verbal and non-verbal expressions.

In the classrooms we will video record different kinds of dialogues between the teacher and the pupils and between the pupils. We will also video record the pupils' work with tasks and the teachers' responses. Through a PRAKUT supported study we explored how we could catch first graders' working processes by using two video cameras (Nilssen et al., 2012). With a handheld camera we followed selected pupils' individual work while another camera recorded the peer collaboration. In the analysis we could watch the recordings simultaneously. In the proposed study we will develop further this methodology, adapting the use of cameras to the foci for the specific lessons. The discussion following each recorded lesson we will based on explicit analytical and conceptual foci in accordance with the research questions and the theoretical framework. Thus a clear purpose and an agenda for discussing, analysing and interpreting the video recordings will be developed (Brophy, 2008; Klette, 2009; Munthe, 2006).

In the video analysis we will draw on interpretative videography, a mix of conversational analysis, ethnography and Goffman's interactional theories on situated expressions (Fuglseth, 2010; Knoblauch, 2008). Videography is well suited to study social and culture based interactions between teachers and pupils in order to catch the constitutive significant elements in the different situations. Using various analytic techniques based on an abductive grounded theory approach (Alvesson & Sköldberg, 2008) the aim is to identify particular pupil and teacher behaviours. Both communicative and semiotic aspects are of interest. For example, in the analysis of the teacher-pupil dialogues we will study how different kinds of questions may support pupils differently and how teachers respond to and extend pupils' contributions. Focusing on pupils' communication in peer groups we will study how and to what extent they support each other, how they reason, argue, share knowledge and establish meaning, and what language and concepts they use.

Rathgen (2006) has shown that joint video discussions and analysis contribute to teachers' professional learning and change of their practices. Sherin (2004) and Coles (2013) demonstrate that they foster the teachers' capacity to direct their attention to pupils' thinking and learning. Teachers tend to talk in a more focused, in-depth, and analytical manner about specific issues related to teaching and learning in these kinds of discussions (Borko et al., 2008; Coles, 2013). In this study we draw on the idea presented by Sherin and van Es (2009) that the ability to observe what is happening in a classroom is a key characteristic of professional vision. Having teachers watch videos from classroom situations provides an opportunity to investigate the points at which teachers pause and comment on aspects which attract their attention. In doing so, we are interested in whether and to what extent teachers notice aspects of particular importance to pupil learning and to the representation of subject matter (Borko et al., 2008; Sherin & van Es, 2009). The research approach of stimulated recall generally involves the replay of teachers' lessons in order to formulate what they thought at the time. Haglund (2003) questions if this method gives access to the teachers' thoughts in the recorded action or rather reflects thoughts created in the interview situation. In our project the last point is relevant, and will be paid attention to during the analysis of the video recordings. The post- and prediscussions of the lessons will be recorded.

Additional data material will be pupils' written work and teachers' written responses. Teachers and researchers will keep diaries to record experiences and reflections. The data material will be managed in NVivo, computer software that facilitates sorting and coding of visual, auditive, as well as textual data.

There are numerous examples of using video-based pedagogy in methods courses in teacher education, but few examples of this kind in mentoring and learning through field experiences (Brophy, 2008). Seidel et al. (2011) conclude that it makes sense to start professional development activities by working with videos of one's own teaching. Central aims for student teachers' field experiences are in accordance with the project's objectives;

developing communicative proficiency, and knowledge about pupils' language and concepts. We will therefore involve student teachers in one of the project schools. By involving student teachers the mentors model how teachers explore and reflect on practice as part of their professional work as teachers. This will also give the student teachers access to teacher educators' research and development projects as proposed by official documents (e.g St. meld. nr. 11, 2008-2009, *Læreren, rollen og utdanningen*).

5. Organisation and project plan

User participation. The project will run from 01.08.14 to 31.07.18. In a preparatory semester the participants will develop an intersubjective understanding of the aims and interventions. The importance of anchoring research and development projects within schools through the teachers' own initiatives is imperative and has been made explicit through various PraksisFoU/PRAKUT projects. Based on a Point-of-view analysis (Udir) we will develop a more precise activity plan in close collaboration with the head teachers and the participating teachers at each school. To establish trust and develop supportive, collegial, reflective and analytical partnerships at the schools is another important issue in this semester. The participating teachers need to be comfortable with being video recorded, and to regard the videos as a means for learning. We also need to develop communication norms and structures for how to explore, analyse and interpret visual data (Borko et al., 2008). The refinement of the project design will be discussed with the reference group.

The interventions and the data collection in schools will run throughout semesters 2–5 consisting of one or two recurring cycles. In each cycle the teachers and the researchers will plan and develop two to three lessons and learning activities in accordance with the objectives and theoretical framework. As part of the planning process the teachers will be asked to imagine how they would like the pupils to respond to given tasks, and how they will assess the pupils' work according to the goals. Later, the video recorded lessons will be discussed and analysed from different perspectives in a collaborative process. We aim to explore and understand pupils' meaning making and development in mathematics, and explore how the pupils' work connects to the teachers' introduction and feedback. The video recorded teaching and learning dialogues will be used to foster conversations about how to improve teaching and learning of mathematics. Pupils' written work and assignments will also be part of the discussions. As told, we will collect and analyse data from all aspects of this process according to our research questions. In order to examine long term impacts of the interventions we will revisit the same classes in semester 7 to gather additional data on communication patterns and mathematical reasoning in the classroom.

The two project schools, Lilleby and Charlottenlund, differ in size. The first is rather small, allowing the teachers in Year 1-4 to work closely together. The latter has three parallels in each year and thus three to four teachers work closely together in each year. Lilleby has a more multicultural group of pupils than Charlottenlund. Both schools have been involved in development projects with HiST focusing on literacy in early grades, and have recently desided to strengthen their education in mathematics. The difference in size and collaborative patterns will affect how we organise the project in the two schools. All teachers in the project will regularly be invited to meetings to share experiences and discuss preliminary findings. Joint one-day seminars for all teachers at the two schools will be organised.

Interventions, data collection, analyses and dissemination will run as parallel activities throughout the project period and will end with an open conference, and production of videos designed for teachers and teacher educators. The participating teachers will take part in the dissemination, by sharing experiences at user-oriented conferences and as co-authors. All researchers will take part in all parts of the project, but their research focus and roles in the interventions will differ according to different competencies. A PhD fellow will develop

his/her own research questions within the main objectives of the project and will be part of the LaUDiM research community and NAFOL.

Student teacher involvement. Two groups of student teachers will be involved in the project, one in semester 3 and one in semester 5 while doing their second year field practice at Charlottenlund. They will study how the participating teachers, who also are their mentors, use video analysis as a learning tool, and experience the same process on their own teaching. Master students will be invited to take part in the project and e.g. to do their master project on data from the project.

Project management and organisation. The interdisciplinary research group represents scholars from mathematics education and pedagogy (education). The project will be managed by Associate Professor Vivi Nilssen, HiST who will also head a steering committee with representatives from the participating schools and HiST. The steering committee will meet regularly to discuss the development and the realization of the project. The interdisciplinary research group will be supported by an international reference group with participants from universities in Sweden, USA, England, and Norway. The reference group is especially important for this project since the research is based at one institution of higher education only. The representatives from the Norwegian Centre for Matematics Education and from University of Agder (UiA) are important links to key competence in mathematic education on the national level. We regard UiA's development work on mathematics in kindergartens as an important point of reference to our project. Bjuland both represents a link to TasS, a relevant PRAKUT project at University of Stavanger on lesson study and teacher education, and brings in his personal competence on mathematics education. Borko who takes part in the TasS reference group together with Nilssen, and also is part of the reference group in CATE (UiO, NFR funded), brings in relevant competence on video analysis and teachers' professional learning. Skott and Jaworski both represent the forefront in mathematics education and teachers' professional learning. Jaworski was formerly the leader of Teaching Better Mathematics (NFR funded), where HiST took part. Research design, data analysis, outcomes and dissemination will be regularly discussed in the reference group.

The participating researchers, institutions, and their roles in the project (CVs enclosed):

Name	**Institution/subject affiliation**	**Role in the project**
Ass. professor Vivi Nilssen	HiST/Pedagogy	Project leader, researcher, and head of steering committee
Professor Frode Rønning	HiST/NTNU/Mathematics education	Assistant project leader, researcher, and member of steering committee
Ass. professor Heidi Dahl	HiST/Mathematics education	Researcher
Ass. professor Torunn Klemp	HiST/Pedagogy	Researcher
Ass. professor Heidi S. Måsøval	HiST/Mathematics education	Researcher
PhD Fellow (75 %)	HiST/Mathematics education	Reseacher
Head teacher Gunn Trøan	Charlottenlund primary school	Member of steering committee
Head teacher Sissel T. Busch	Lilleby primary school	Member of steering committee
4 teachers from the 2 schools		2 members of the steering committee
Research assistant	HiST	Support
Professor Hilda Borko	Stanford University	Member of reference group
Professor Barbara Jaworski	University of Loughborough	Member of reference group
Professor Jeppe Skott	Århus University/Linnæus University	Member of reference group
Professor Raymond Bjuland	University of Stavanger	Member of reference group
Ass. professor Martin Carlsen	University of Agder	Member of reference group
Director Kjersti Wæge	Norwegian Centre for Mathematics Education	Member of reference group

Project plan

No.	Main activitiy, goals and deliverables	Total expenses	Responsible partner	Participating partners
1	Communication with participants Seminars and meetings with teachers, steering comitte, reference group. Network meetings	1750	HiST	Charlottenlund Lilleby
2	Data collection in schools, year 1-4	1820	HiST	Charlottenlund Lilleby
3	Datacollection teacher education	850	HiST	Charlottenlund Lilleby
4	Analysis of collected material	5960	HiST	
5	Dissemination, Scientific and user-oriented conferences and journals. LaUDiM conference, and video production.	5950	HiST	User-oriented: Charlottenlund p. s. and Lilleby p. s.

Please consult the attached project plan for more detailed information.

6. Key milestones

No.	Time	Activity
1.	Aug -15	Evaluate and revise the classroom interventions in dialogue with the participating schools.
2.	Jan -16	Evaluate and revise the classroom interventions and the interventions in teacher education with participating schools. Evaluate inteventions and the first analyses with the reference group.
3.	Aug -16	Evaluate and revise the classroom interventions in dialogue with the participating schools.
4.	Jan -17	Intervention phase terminated.
5.	Des -17	Data collection phase terminated.

Please consult the attached project plan for more detailed information.

7. Costs incurred by each research performing partner (NOK 1 000)

Main activity, objectives and deliverables	Research performing partner	Payroll and indir. exp.	Equipment	Other op. expenses	Totals
Communication with participants	HiST	1410		340	1750
Data collection in schools	HiST	1485	100	235	1820
Datacollection teacher education	HiST	565	85	200	850
Analysis	HiST	5850		110	5960
Dissemination	HiST	5200		750	5950

8. Financial contribution by financing partner or other user (NOK 1 000)

Financing partner	Cash financing
HiST	9387
From Research Council	6969

9. Other collaboration

Informal contact with Paul Andrews and Judy Sayers, Stockholm University; Ola Helenius and Görel Sterner, NCM, Göteborg. Barbara Jaworski and Hilda Borko are both prepared to stay at HiST as guest researchers in 2016/2017. Vivi Nilssen is also welcome to stay at Stanford University as a guest researcher.

PART 3: Project impact

10. Impact for national knowledge base

Impacts factors in LaUDiM: 1) Developing "best practice" in early years mathematics with a special emphasis on mathematical reasoning and arguing, 2) Developing a higher understanding of the connection between the *milieu* in the mathematical classroom and pupils' learning outcomes, 3) Developing "best practice" in video-supported professional learning and learning for student teachers in the field practice, 4) Developing methods for users to be part of research in ways that improve the practice field, and 5) Producing video material to be used in professional development.

11. Relevance for the partners and the society

As outlined in the letters of intent, LaUDiM connects with the two schools' documented need to increase their focus on early years mathematics.

LaUDiM addresses important issues concerning national focus areas: 1) Increasing the achievement level in mathematics, 2) Developing basic skills in mathematics for all (LK06), 3) Strengthening the professional relevance, the interdisciplinary collaboration and the researched based teaching in Teacher Education (GLU 1-7), and 4) Strengthening teachers' competence in mathematics through school based development and further education.

PART 4: Other aspects

12. Communication of results

We plan to communicate results from the project through scientific and user-oriented journals and conferences, both national and international (please consult the application form for details). In spring 2018 we will organize a user-oriented scientific LaUDiM conference. In addition we will produce videos focusing on classroom dialogues for use in Teacher Education.

13. Environmental impacts

The proposed project will not affect the external environment.

14. Ethical concerns

The proposed project will be carried out according to NESH. All participation will be voluntary (informed consent) and anonymity will be taken care of.

15. Gender issues

Of five researchers in the project, four are females. One of them is qualifying for full professor in the near future and the other three in the coming years. The project group will be directed by a female researcher. The lack of gender balance in the research group will to some extent be taken into concideration when we recruite the PhD fellow and the participating teachers/menthors. However, the reference group is well balanced.

16. Additional information specifically requested in the call

The proposed project answers FINNUT's call for practice based innovative intervention studies based on users' need, and on collaboration between researchers and practitioners throughout the whole research process.

Selected references

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