

A classroom study of young pupils' work with mixed numbers

Siri-Malén Høynes, Pernille Friis, Heidi Dahl

ECER, Dublin

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LaUDiM – intervention project supported by FiNNUT (Norwegian research council)

Intervention project together with elementary schools where we explore and develop teaching and learning in mathematics.

→ Develop knowledge on how the learning environment in the early years of elementary school influence development of pupils abilities to express mathematical concepts and mathematical ideas.



Some goals of the project

- Improve pupils' proficiency in mathematical reasoning, arguing and justification.
- Improve pupils' proficiency in expressing mathematical concepts and ideas using a variety of representations.

Research question

How do young pupils make sense of different semiotic representations of mixed numbers?

Different kinds of semiotic representations:

1. Numeric symbols
2. Sentences written in natural language
3. Figures

Examples

1. $3\frac{1}{2}$

2. '3 and a half'
'half of 7'

3.



Data material

Transcribed [video recordings](#) from 2 lessons in a 2nd grade classroom

- Some pupils working in pairs and groups of 4
- Teacher leading a discussion with the class after they have worked on the problem

Participants

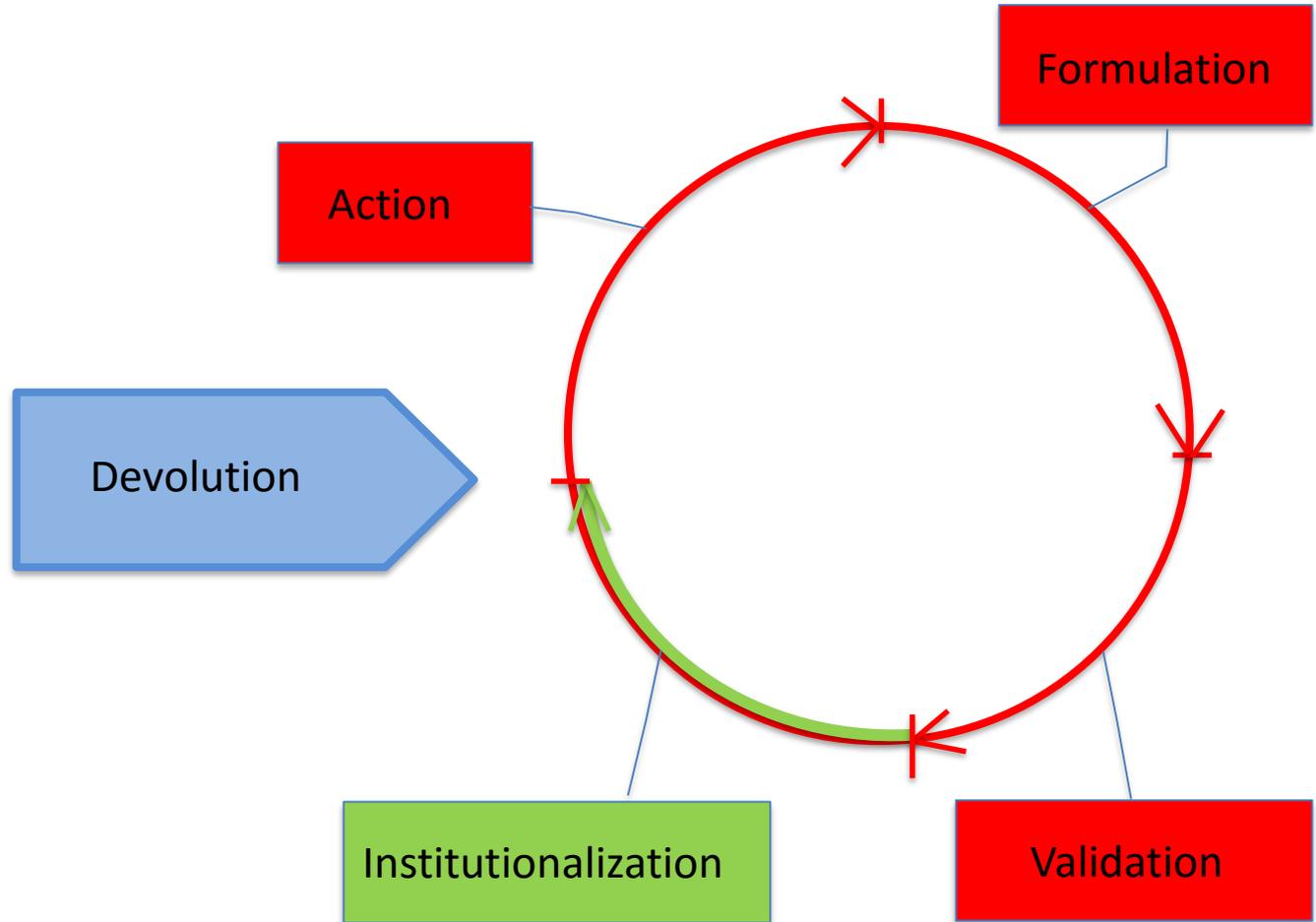
- A Norwegian 2nd grade class
- Teacher – with masters in mathematics didactics
- Research group observing and interacting with pupils

The teacher and the researchers co-planned the lesson.

Designing the tasks in the lessons

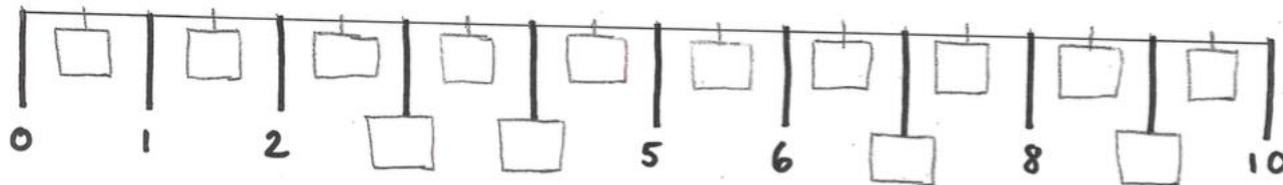
- inspired by TDS

Analysis of the target knowledge and task design

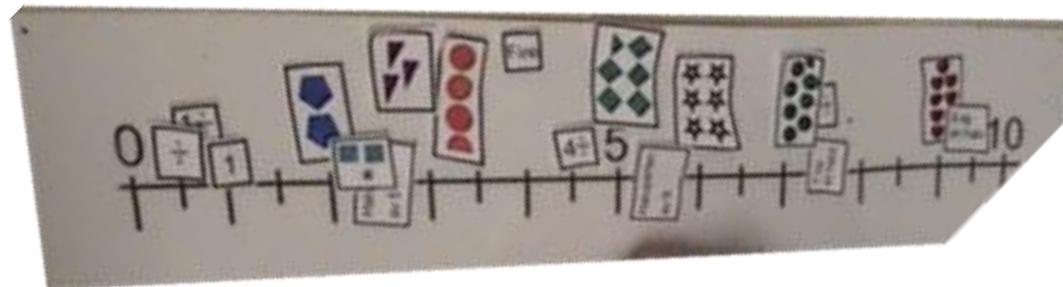


The lessons

1. Two tasks in the first lesson
 1. Divide an odd numbered set of objects equally between two people and write how much each of you got on a piece of paper.
 2. Fill in missing numbers on a ruler



2. Place different semiotic representations of mixed numbers on a number line



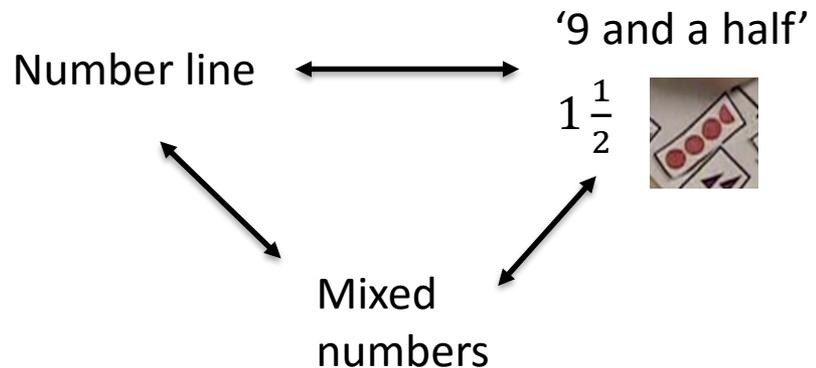
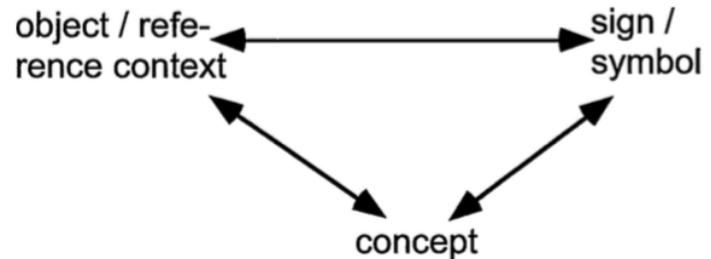
Analysis

Two steps:

1. Identify all the excerpts in the data material where we hear pupils reasoning *“The term reasoning is defined as the line of thought, the way of thinking, adopted to produce assertions and reach conclusions. Argumentation is the substantiation, the part of the reasoning that aims at convincing oneself, or someone else, that the reasoning is appropriate.” J. Lithner (2000)*
 - Included all reasoning, not only argumentation
 - Disregarded reasoning strongly influenced by researchers
 - No excerpts found in the material from the first lesson.
2. Analyse excerpts using Steinbring’s epistemological triangle
 - In this presentation we look at one excerpt from the institutionalization in lesson 2.

Steinbring's epistemological triangle

- Inspired by Ogden and Richards' triangle of meaning
- Models how we give meaning to signs and symbols through mediation between the sign/symbol and the object/reference context.



Results

- Many pupils struggled to use the number line
 - Counted all markings on the number line as a whole
- Figural representations are in most cases placed correctly (according to the pupils' understanding of the number line)
- Symbolic and written sentences provide the biggest challenges



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Sentence: '9 and a half' (part 1)

Tina: Where did you find out that '9 and a half' should be?

Paul (moves the box): It should be there (places it correctly)

Paul: No it wasn't going to be there, it was going to be THERE (points to 8.5).

Tina: Yes that is what I thought I remembered.

Paul: Nine and a half should. 9 and a half. Half is before whole.

Tina: (helps move the box) It was here your thought it should go?

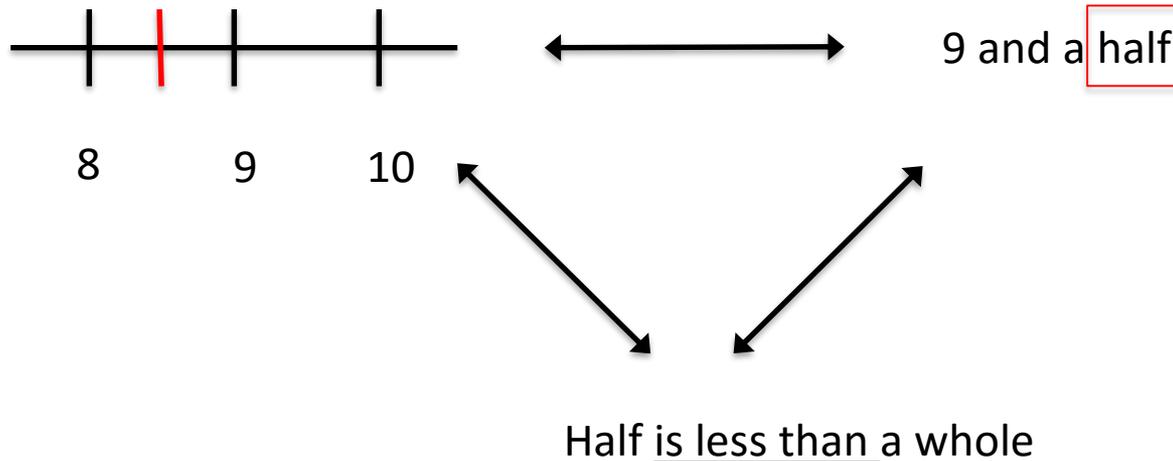
Paul: Yes

Tina: I heard many groups talking about this. How do we know, nine and a half, is nine and a half, should it be before or after 9? (silence) Because I think you moved that around a bit didn't you Patrick and Paul? Why did you move it?

Patrick or Paul: Because, really it shouldn't. Really, a half it is right before a whole. So then it is a bit different. Then it is, if it is, if it is, if you and I, or, did like the nine went on the ten, on the one right before ten. Then it is a bit weird. Then it is not a whole nine, no it is not a whole ten. Then there is a whole ten after that.

Analysis of the excerpt

Paul: Nine and a half should. 9 and a half. Half is before whole.





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Sentence: '9 and a half' (part 2)

Tina: And this is a very, and this is a very important thing we need to figure out now. Nine and a half, is that before or after nine? If we think that this is nine (points to 9). Right Patrick?

Tina: But listen. The question that Paul and Patrick discovered. That is very important and we need to figure it out! Nine and a half, should it be here, before nine, because now nine is here, should it be before nine or after nine? Who has figured out something to share? Peter?

Peter: It is after nine, because it isn't less, or, minus a half it is AND a half.

Tina: So you mean that nine and a half should really go here? Explain again why you think that.

Peter: Nine and a half, it is AND a half, it isn't MINUS a half. So it is not before nine, it is after.

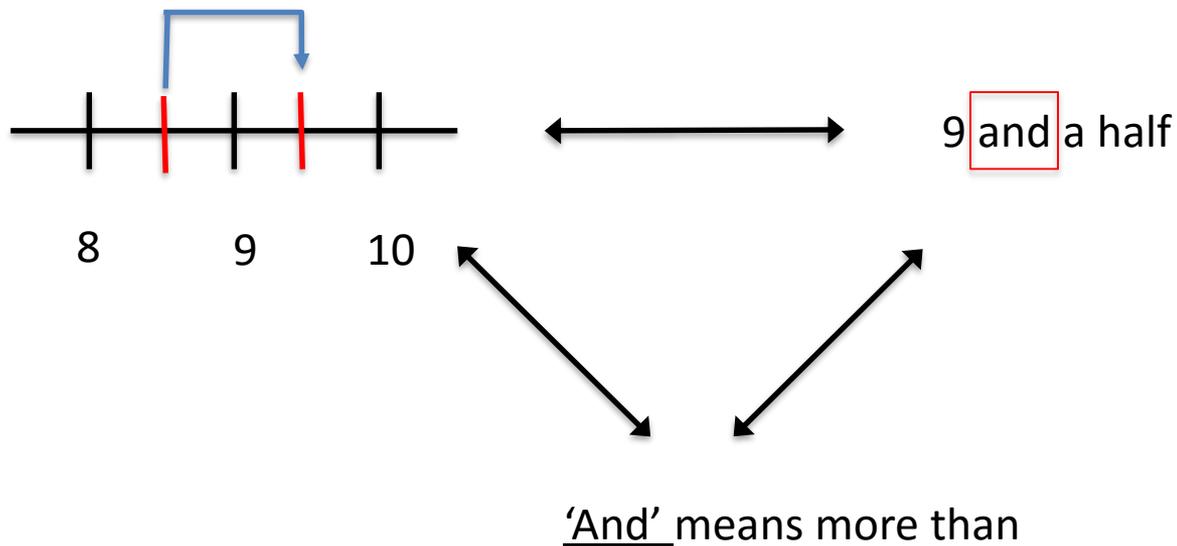
(later in the discussion)

Tina: This is difficult, how can we figure it out? Paul, what have you found out?

Paul: Me and Patrick actually thought a bit wrong, 9 and a half SHOULD really be there (points to 9.5). Since then it is a bit weird, become, shall, since, if it is 9 and a half, then it is a half, and a half MORE, not a half less.

Analysis of the excerpt

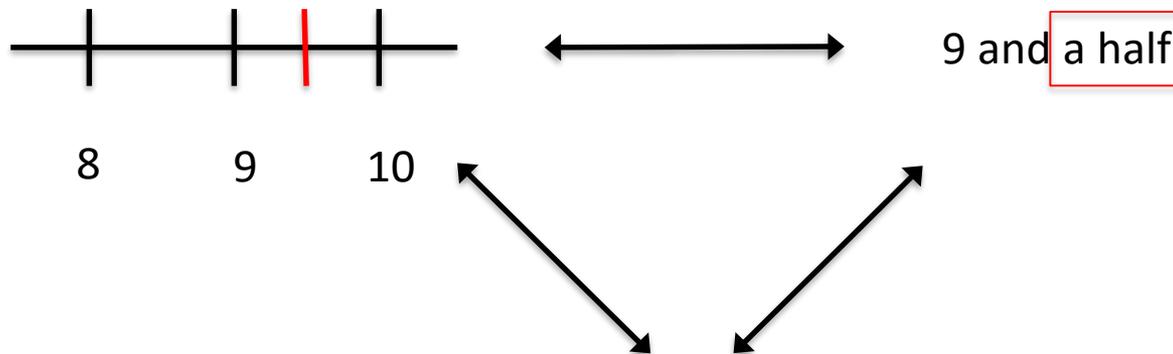
Peter: It is after nine, because it isn't less, or, minus a half it is AND a half. (...) Nine and a half, it is AND a half, it isn't MINUS a half. So it is not before nine, it is after.



Analysis of the excerpt

Peter: Nine and a half, it is AND a half, it isn't MINUS a half.
 (...)

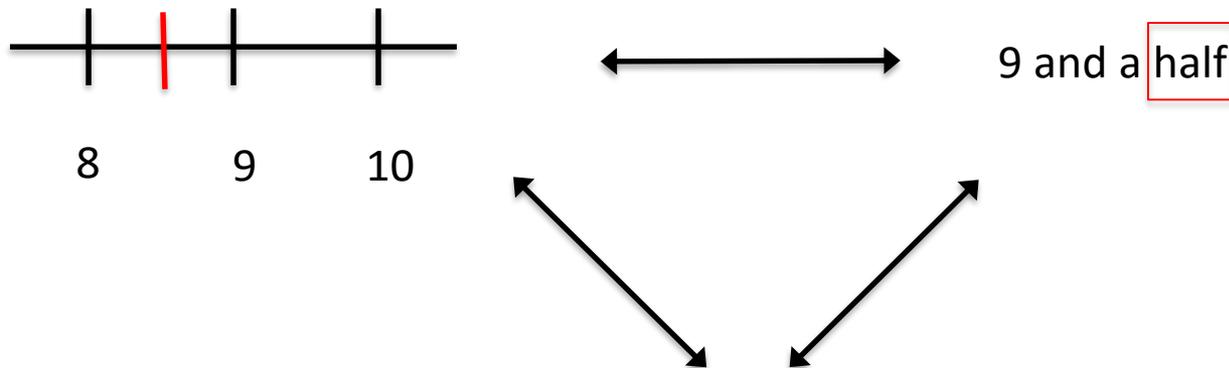
Paul: If it is 9 and a half, then it is a half, and a half MORE,
 not a half less.



'And' means more than

'A half' is a quantity that can be added or subtracted to any number

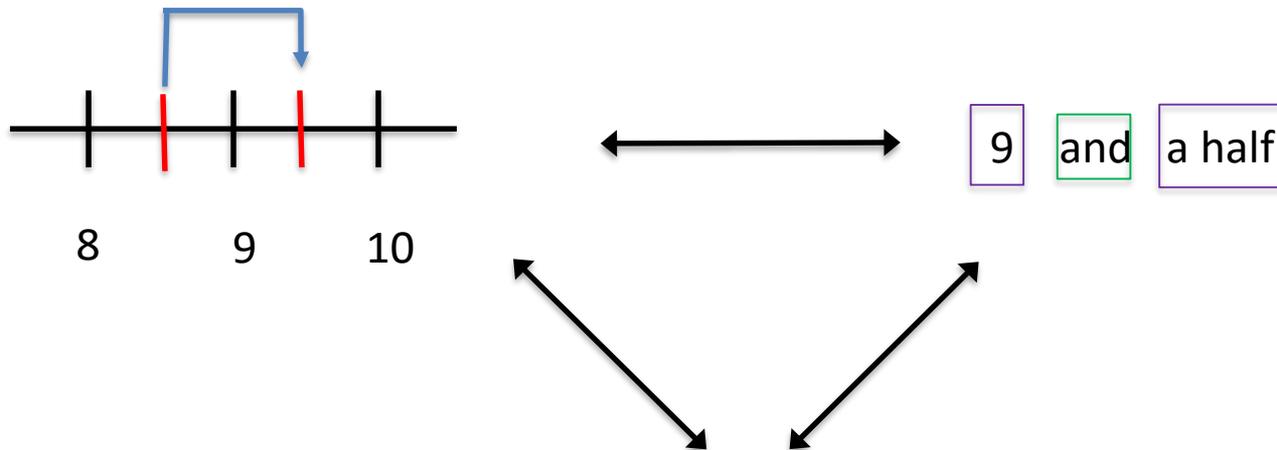
Analysis of the excerpt



Mixed numbers:

- Half is less than whole

Analysis of the excerpt



Mixed numbers:

- Half is less than whole
- And – more than – addition
- A half is a quantity that could be added or subtracted

Discussion

- Were the pupils ready to use the number line as a reference context to make sense of the mixed number representations?
- In the Norwegian way of telling time "Halv ni" (Half nine) is 8:30. Could this be a context the pupils use to conclude that "half is less than whole"?
- In many cases the pupils saw no need to argue their placement of the representations. How do we make them more eager to reach an agreement when they have different solutions?



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