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A classroom study of young pupils' work with mixed numbers.

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Proposal information

The project presented here is part of a four-year intervention project where we collaborate with two primary school teachers to create a successful learning culture in early learning of mathematics, with special emphasis on language development. Two of the main objectives of the project is to improve pupils' proficiency in expressing mathematical concepts and ideas using a variety of representations, and to improve their proficiency in mathematical reasoning, arguing and justification. Recent research shows that mathematical reasoning is important for children's later achievement in mathematics (Nunes, Bryant, Sylva & Barros, 2009). There is thus a need to conduct research on how young pupils learn mathematics, especially through language use and reasoning, and how they understand and use mathematical concepts and symbols in these processes.

Young children can benefit from early introduction to the concept of fractions, before they develop an ingrained theory of numbers that is restricted to counting numbers (Siegal & Smith, 1997, p. 18). While others have analyzed how young children make notations for fractions to gain insight in children's learning of fractions (Brizuela, 2006), we have investigated how children reason while trying to make sense of fractions and mixed numbers using a variety of representations. The research question we focus on here is:

How do young pupils give meaning to semiotic representations of mixed numbers?

In mathematics, the use of semiotic representations is crucial. We need semiotic representations both to think about and explore mathematical concepts, and to communicate mathematical ideas. All mathematical objects are abstract in their nature, and we only gain access to them through semiotic representations (Duval, 2006). For learners of mathematics, this gives rise to a potential cognitive conflict; how can the mathematical object be distinguished from the representations at use, if the use of semiotic representations is the only way to get access to the mathematical object (Duval, 2006, p.107)? The ability to interpret semiotic representations and to change between different representations is thus a critical threshold for the development of mathematical understanding.

As a theoretical framework we use semiotic theory as described by Steinbring (2005, 2006). Steinbring views mathematical signs as "instruments for coding and describing mathematical knowledge, for communicating mathematical knowledge as well as for operating with mathematical knowledge and generalizing it". He emphasizes that all mathematical signs have both an epistemological and a semiotic function (Steinbring 2006, pp.133-134). Thus, a sign can be thought of as representing an abstract mathematical concept as well as a concrete object or reference context (Rønning, 2013, p.162). Learning in this framework is seen as meaning making through mediation between the sign/symbol and the object/reference context. This process is captured in Steinbring's epistemological triangle (where the third corner is the concept) (Steinbring, 2006, p.135). One can see the sign/symbol corner as containing the signs that the learner tries to give meaning to, and the object/reference context corner as containing the actual situation that is represented by the signs, i.e. the context that the learner uses to give meaning to the mathematical sign. The mathematical concept mediates between the sign/symbol and the object/reference context. If there is no mediation through a concept, there is just an associative connection between the sign/symbol and the object/reference context (the sign's semiotic function). It is worth noticing that the epistemological triangle is not a static system that is independent of the learner. The connections between the three corners have to be actively constructed, often in interaction with others (Steinbring, 2006). When the learner's mathematical knowledge develops the reference context will tend to become less concrete (Steinbring, 2005, p. 30).

Methods

In order to address the intervention project's research questions we have chosen a video-based design which promises to be an important tool in intervention studies (Munthe, 2006). Video-design encompasses the complexity and diversity of voices, perspectives and issues at play during teaching and learning in classrooms, and makes it possible to freeze, capture and recapture in detail situations in teaching and learning processes (Klette, 2009).

The main data for this paper are video and transcripts from two lessons from a 2^{nd} grade classroom, where the pupils worked with mixed numbers. The two lessons were planned in collaboration between the teachers and the research team. Three tasks were given to the pupils. The first task was to divide a set of some figures equally between two people and the second task was to fill in names for the marks half way between whole numbers on a number line. The last task was to place images representing different mixed numbers on a number line. These images were of three types: figures (such as three and a half circle), numeric symbols (such as $6\frac{1}{2}$ (written with a horizontal fraction bar)), and sentences in natural language (such as «9 and a half»). In the lessons, the pupils worked both in pairs, in groups of four and participated in whole class discussions.

In our analysis, we first identify all the episodes in the data material where the pupils attempted to explain their reasoning, since we are interested in how pupils give meaning to the semiotic representations of mixed numbers. Inspired by discourse analysis we identify which ways of reasoning about mixed numbers that were available to the class (Marshall, 1994, p.92). We identify a turning point in the dialogue when a pupil gave a valid argument for the correct placement of "9 and a half", and made this way of reasoning available to the rest of the class. In this presentation, we will focus on the episode containing this turning point. This episode is analyzed further using the epistemological triangle of Steinbring (2005, 2006).

Expected outcomes/results

The preliminary analysis indicates that the pupils have different interpretations of the different semiotic representations of mixed numbers. Based on the reasoning made by the pupils in the whole class discussion, the pupils seem to have a better understanding of the figural representations of mixed numbers than the other representations.

The data show that making sense of the natural language representation of a mixed number is very challenging for the pupils. This difficulty is present both when the pupils work in small groups and in the classroom discussion at the end of the lesson, when the teacher was focusing the discussion on whether “9 and a half” is before or after 9 on the number line. The turning point in the discussion occurs when one of the pupils sees that “9 and a half” is actually “9 AND a half” and links “and” to the operation addition. He then concludes that it cannot be less than 9. Here the pupil now sees that the word “and” carries significant meaning in the sign “9 and a half”. This suggests that the original misconception, that “9 and a half” is before 9, is linked to the children’s preconception that half is less than whole. They put emphasis on the word “half”, and thus interpret “9 and a half” as being less than 9.

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Intent of publication

The presentation will be developed into an international article, journal not decided.