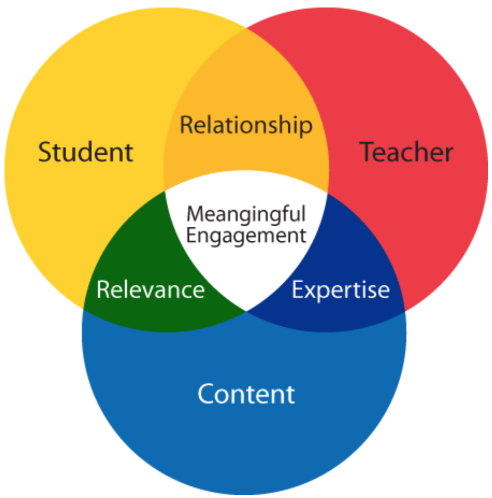
Designing mathematical tasks in LaUDiM

- aiming at pupils' use of particular pieces of mathematical knowledge



Seminar in honour of Geir Botten

NTNU (FLT) — 26 May 2016

Heidi Strømskag

NTNU (IMF)

PRESENTATION

- The *knowledge* and a *situation* that is intended to make it meaningful
  - Rooted in the theory of didactical situations (TDS)
- Halving a set of elements
  - One-half as part of a whole (multitude — magnitude)

TDS —The theory of didactical situations

- Scientific approach to the PROBLEMS problems posed by the teaching and learning of mathematics, where the *particularity of the knowledge taught* plays a significant role

TDS — Knowledge

"For a scientific mind, all knowledge is an answer to a question. If there has been no question, there can be no scientific knowledge."

(Bachelard, 1938/2001, p. 25)

TDS — Methodological principle

A particular piece of mathematical knowledge can be represented by a SITUATION that involves problems that can be solved in an optimal manner by *using* this knowledge

- the necessary *conditions* for a situation to implement the knowledge aimed at
- how a situation can be *designed* and its evolution managed

TDS — The milieu

- The *intellectual and physical reality* in which students operate when solving a problem
- Is relative to a piece of mathematical knowledge and consists of:
  - The task (problem) to be solved
  - Material and symbolic tools (e.g. technical tools, manipulatives, informative texts, data)
  - Students' prior knowledge
  - Interaction:
    - *Rules to operate in the situation*
    - *Who is supposed to interact with whom—and how?*

TDS — FEATURES

- GOAL: Model relationships between teaching and learning through a systemic approach in order to make controlled actions on teaching (Artigue, 1994).

- ORIENTATION: TDS puts mathematical knowledge at the centre of analysis of classroom interaction

- LEVEL OF ANALYSIS: *Macro* perspective → activity is analysed from the perspective of the implicit expectations and constraints imposed by *the institution*

PRESENTATION

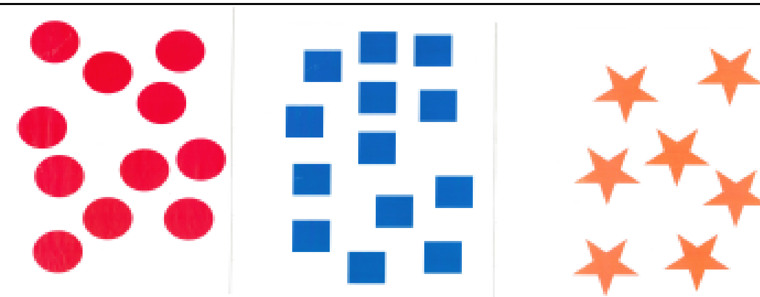
- Some *target knowledge* and a *situation* that makes it meaningful
  - Rooted in the theory of didactical situations (TDS)

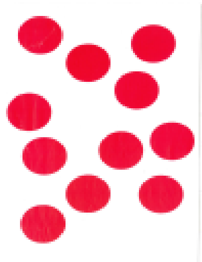
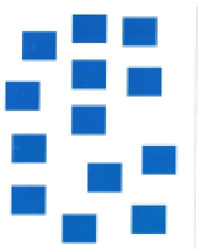
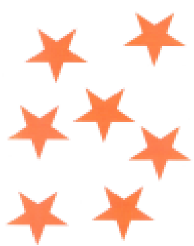
- Halving a set of elements
  - One-half as part of a whole (multitude — magnitude)

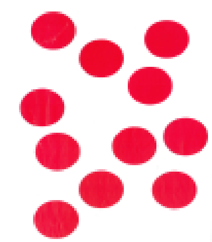
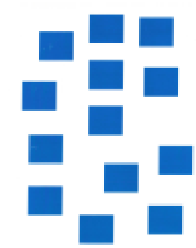
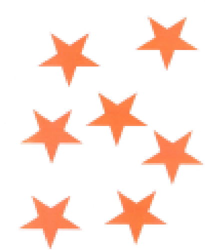
CLASSROOM EPISODE

- Charlottenlund school
- Third Grade
- 14 pupils
- Teacher Oda
- Lesson has been planned by Oda and researchers (Frode, Heidi D, Heidi S, Torunn, Vivi)

Halving a set of elements (Part I)

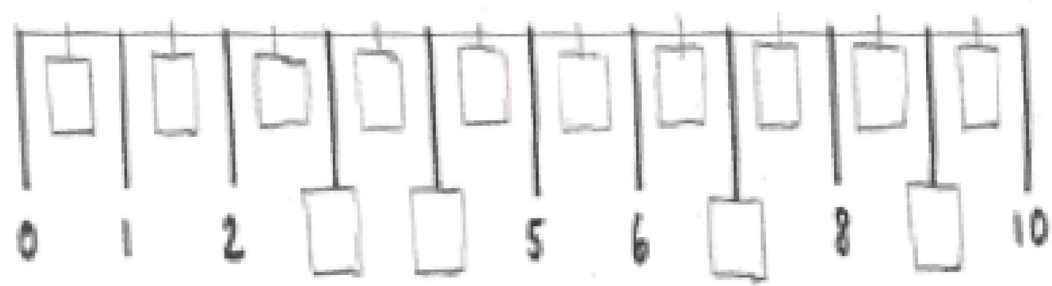
Charlottenlund school — 19 Jan 2015		
Target knowledge	Use of knowledge	Material milieu
1) Halving of a set of an odd number of elements 2) Writing down the result of the halving (how many)	Distribution of an odd number of elements in two equal parts Comparison (inside each pair) of written record of "how many"	

Mathe-matical topic	Implemented as	Process	Target knowledge	Use of knowledge	Material milieu
Halving → *one-half* as one of two equal parts of a whole	**One-half** (Norwegian: "halvparten")	In pairs: Distribution of an odd number of figures—equally between two pupils (figures are cut out → the odd one is cut in two halves)	1) Halving of a set of an odd number of elements 2) Writing down the result of the halving (how many)	Distri-bution of an odd number of elements in two equal parts Compari-son of written record of "how many"	  

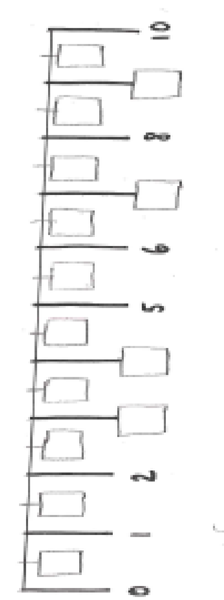
Mathe-matical topic	Implemented as	Process	Target knowledge	Use of knowledge	Material milieu
Halving → *one-half* as one of two equal parts of a whole	**One-half** (Norwegian: "halvparten") 1) One-half as part of a whole: one-half of the *set* of figures →*multitude* 2) One-half as part of a whole: one-half of the *size* (area) of the odd figure →*magnitude*	In pairs: Distribution of an odd number of figures—equally between two pupils (figures are cut out → the odd one is cut in two halves)	1) Halving of a set of an odd number of elements 2) Writing down the result of the halving (how many)	Distri-bution of an odd number of elements in two equal parts Compari-son of written record of "how many"	  

Comment: Because there is no focus on the dividend, the situation is *not* a division situation. Further, because there is no focus on any input-value on which to operate, it is *not* a situation with one-half as an operator. (e.g. Lamon, 2012).

"RULER" (Part II – same day)

Target knowledge	Use of knowledge	Material milieu
Representing decimals in the middle of integers as $\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}$ ...		

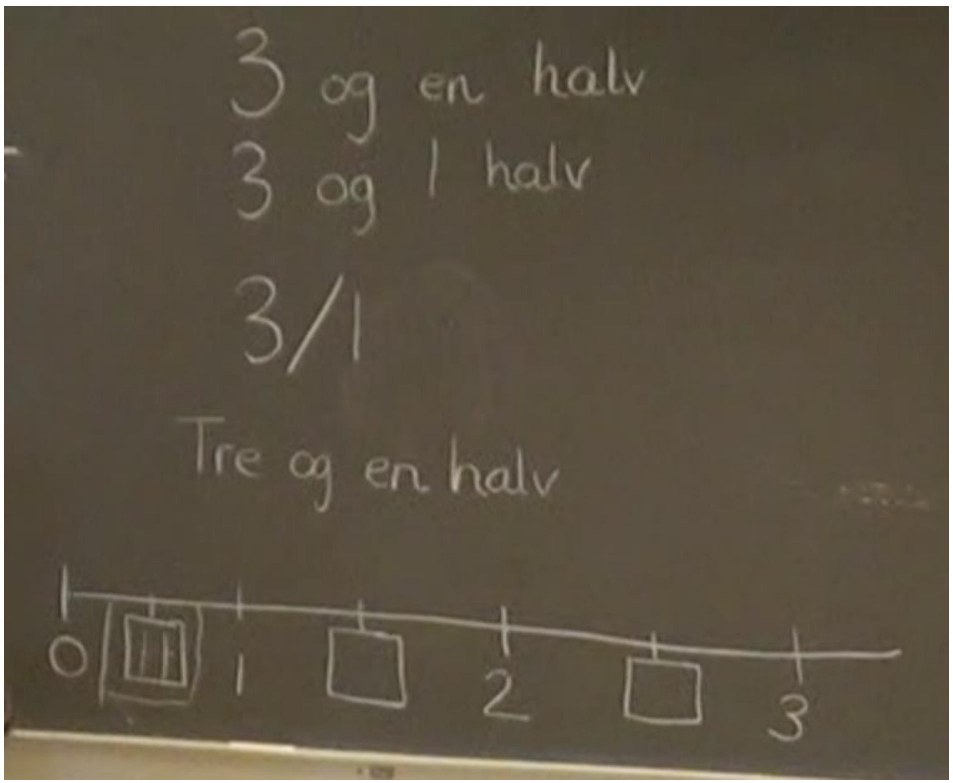
"RULER" (Part II—same day)

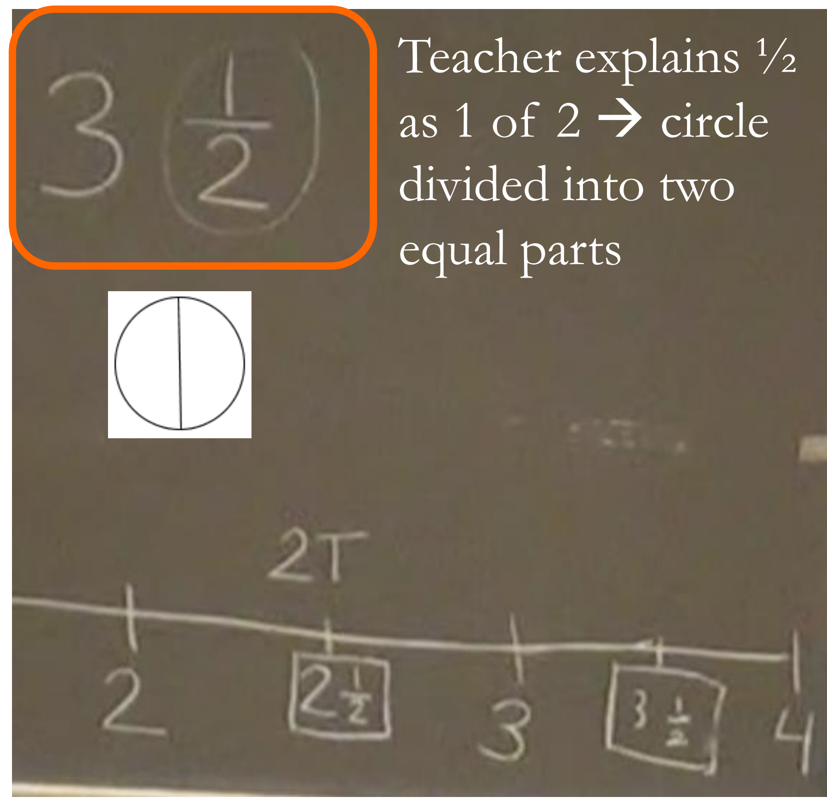
Mathe-matical topic	Implemented as	Process	Target knowledge	Use of knowledge	Material milieu
Decimals in the middle of integers (from zero)	"Ruler" → (partly) empty number line [0,10]	Writing—on an "empty" number line —integers and decimals in the middle of them (from 0 to 10)	Representing decimals in the middle of integers as $\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}$ …		

Comment: Pupils came up with several representations. Institutionalisation of notation by fraction ($\frac{1}{2}$) → institutionalised by the teacher as "one of two", motivated by pupils' cutting of the odd figure in two halves. Illustrated by the teacher as a circle cut in two halves 

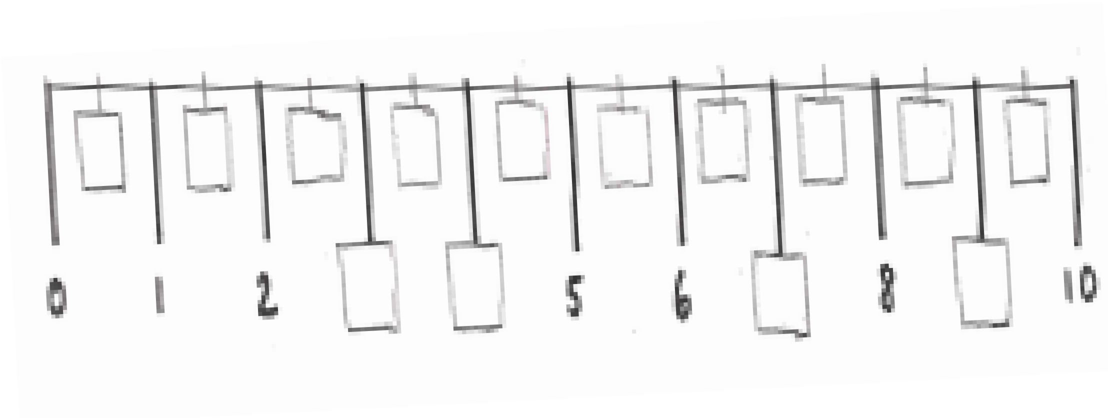
INSTITUTIONALISATION OF $3\frac{1}{2}$

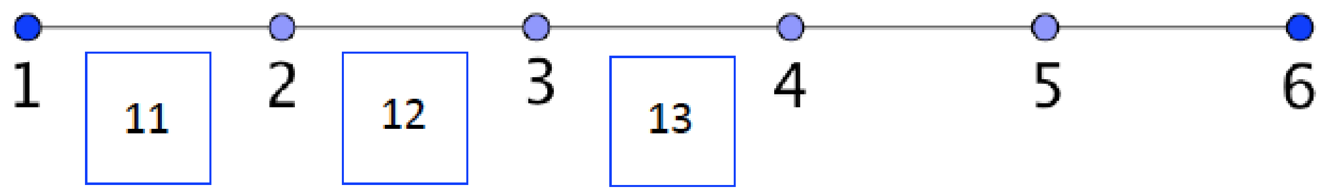
- Building on pupils' solutions:

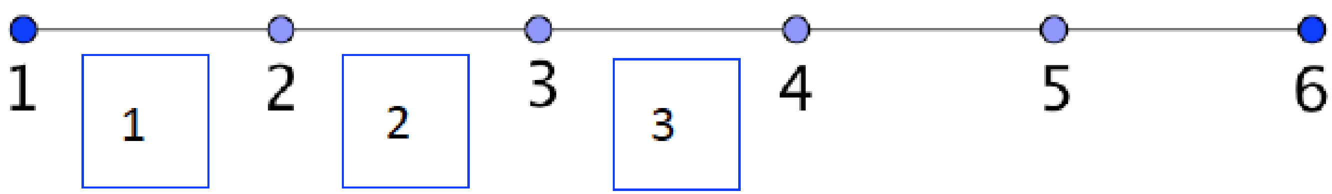




Pupils' solutions — following a pattern

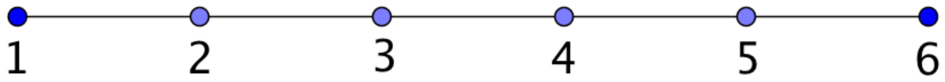






DIFFERENT ASPECTS OF DECIMALS

- 3 ½ on the number line → the number between 3 and 4



- 3 ½ as an amount → half of 7

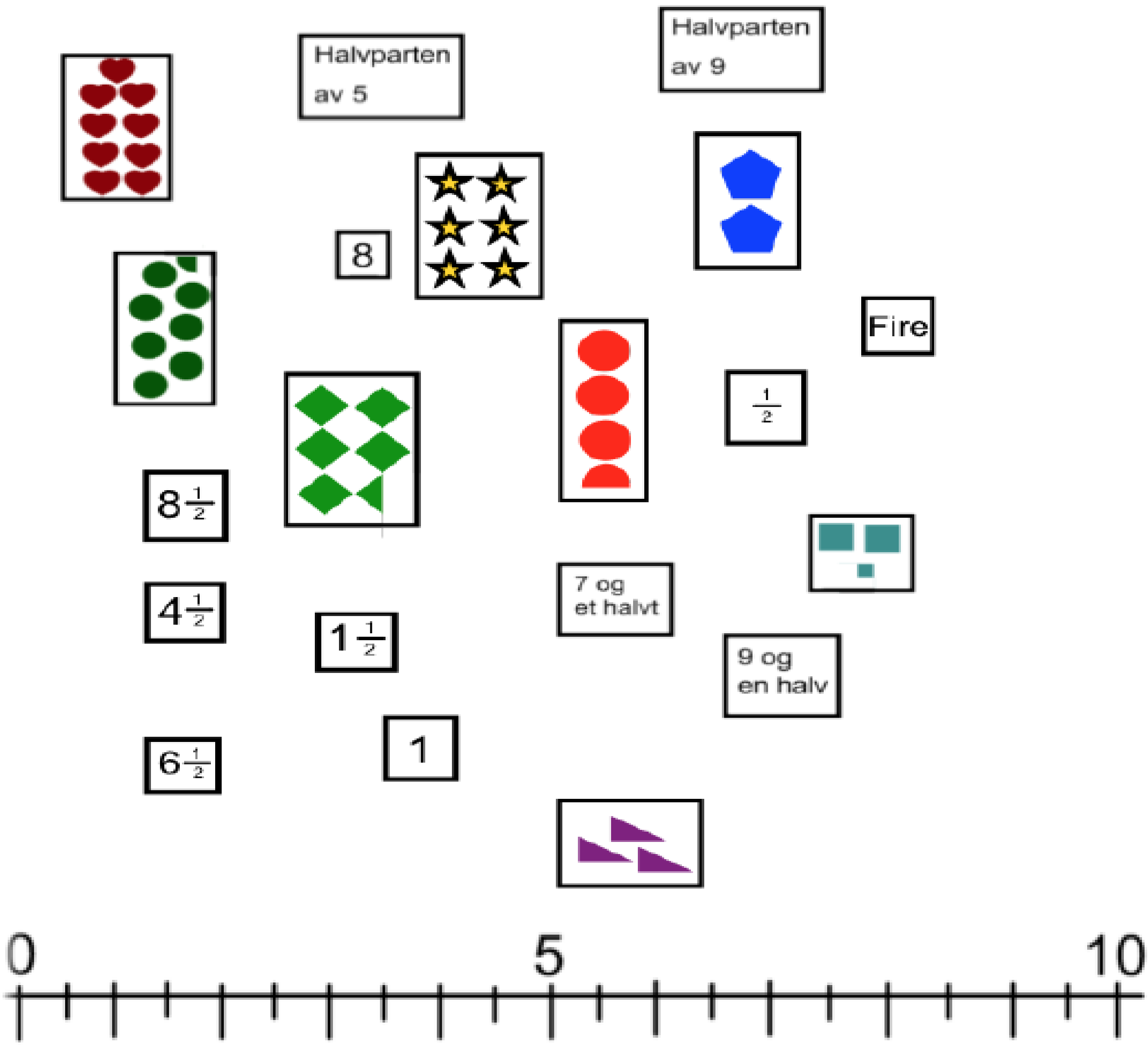
- 3 ½ as part of a whole:
  one-half of six figures → *multitude*
  one-half of the size of the odd (left over) figure → *magnitude*

- 3 ½ figures (half the amount) VERSUS the location of 3 ½ on the number line

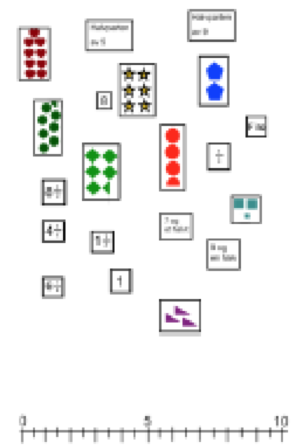
- Two separate activities

- Using the “ruler” to measure → potential feedback from the milieu

PART III – A WEEK LATER



CONNECTING THE ASPECTS

Charlottenlund school — 26.01.2015					
Mathe-matical topic	Implemented as	Process	Target knowledge	Use of knowledge	Material milieu
Decimals in the middle of integers (from zero)	"Ruler" → empty number line [0,10]	Placing—on a number line—different kinds of representations of decimals in the middle of integers [0,10]	Knowing where $\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}$ etc. are located on the number line		
Comment: $1\frac{1}{2}$ denotes one half MORE than a whole → a convention. Cf. conversation in "corner" about the location of "9 and a half" (*before* or *after* 9).					

9 ½ — IS THAT BEFORE OR AFTER NINE?

19. Oda: Hvor fant dere ut at 9 og en halv skulle være da?
20. Fredrik (flytter på boksen): Den skal være der (plasserer den riktig)
(Oda snakker om hvordan det går an å plassere for å få god plass).
21. Fredrik: Nei det var ikke der den skulle være, det var DER (peker på 8,5) den skulle være.
22. Oda: Ja det var det jeg synes jeg huska

23. Fredrik: 9 og en halv skal. 9 og en halv. Halv er jo før hel. (Oda hjelper å flytte).
24. Oda: Det var her dere tenkte den skulle?
25. Fredrik: ja.

26. Oda: Det hørte jeg det var flere som snakka litt om.. hvordan vet vi, 9 og en halv, er 9 og en halv, skal det være før eller etter 9? (4 sekunder pause) For det tror jeg dere flytta om på litt, gjorde dere ikke det Kristian og Fredrik? hvorfor flytta dere på det?

27. Kristian/Fredrik: Fordi, egentlig så skal jo ikke. Egent, en halv det er jo rett før hel. Så da blir det jo litt annerledes. Da blir det jo, hvis det blir, hvis det blir, hvis du og jeg, eller, gjorde sånn at 9eren skulle på 10, på den rett før 10. Da blir det jo litt rart. Da blir det jo ikke en hel 9er, nei det blir ikke en hel 10er. Da blir det en hel 10er etter det.

28. Oda: og det er jo et veldig, og det er en veldig viktig ting som vi må finne ut av her nå. 9 og en halv, er det før eller etter 9. Om vi tenker at det her er 9 (peker på tallinja). Sant Kristian.

33. Johannes: Det er etter 9, for det er ikke mindre eller, minus en halv det er OG en halv.
34. Oda: Så du mener at 9 og en halv skal egentlig være her? Forklar en gang til hvorfor du mente det.

35. Johannes: 9 og en halv, det er jo OG en halv, det er jo ikke MINUS en halv. Så det blir ikke før 9, det blir etter.
36. Oda: Hva var det du tenkte å si Selma?
37. Selma: Ehm, det, 9 og en halv bør flyttes før 10.
38. Oda: Kan du vise hvor? (Selma flytter den til 9,5).
39. Oda: Så du mener den skal flyttes dit?
40. Selma: Ja, fordi ehm, en halv er før på en måte tallet, så viss 9 så er 9 og en halv før 9.
41. Oda: Men nå er jo 9 her da (peker på 9) Selma

53. Fredrik: Jeg og Kristian har faktisk tenkt litt feil, 9 og en halv SKAL egentlig være der (peker på 9,5). Siden da blir det litt rart, bli, skal, siden, viss det er 9 og en halv, da blir det jo en halv, og en halv MERE, ikke en halv mindre.

REFLECTIONS

- Institutionalisation of pupils' ways of writing 3 ½
- Letting pupils explore where to place 9 ½

- The milieu does not offer an adidactic potential for the different aspects of decimals
- Weak feedback potential in existing milieu
- USE of knowledge

DESIGNING TASKS → SITUATIONS

- Mathematical tasks/problems are only one part of the milieu (but an important one)
- Artefacts, symbol use, how to collaborate, using the knowledge

Thank you for your attention!