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MODELLING SITUATIONS INVOLVING EQUAL-SIZED GROUPS

PME 41 – Singapore 2017

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Research project

- The LaUDiM Project (Language Use and Development in the Mathematics Classroom)
  - a four-year developmental research project (Trondheim, Norway)
- Collaboration between:
  - 2 mathematics teachers (in two primary schools)
  - 5 didacticians (teacher educators of mathematics)
  - 2 pedagogues (teacher educators)
- Goal:
  - Developing teaching — aiming at students' (Grades 1-5) mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001)
- Research methodology:
  - Cycles of: planning – designing – implementation – validation

MULTIPLICATIVE STRUCTURES

In a teaching situation, mathematical operations are dealt with not only as calculations, but also in terms of *how they model different situations*

→ Vergnaud (1988): Mathematical concepts are rooted in situations and problems

The most important types of situations where multiplication of integers is involved:

- equivalent groups (e.g., 6 tables, each with 4 children)
- multiplicative comparison (e.g., 3 times as many girls as boys)
- rectangular arrays/areas (e.g., 4 rows of 7 students)
- Cartesian products (e.g., the number of possible trousers-sweater pairs)

(Greer, 1992)

THE NORWEGIAN CONTEXT

In Norwegian schools, multiplication is usually introduced through situations with equivalent groups, where $4 \cdot 7$ means 7+7+7+7, while $7 \cdot 4$ means 4+4+4+4+4+4+4

→ Repeated addition

→ The first of the factors in the product – the number of equivalent groups – is taken as the *operato*r (termed multiplier)

→ The other factor – the size of each group – is taken as the *operand* (termed multiplicand)

OBSERVED SESSIONS AND GOALS

- Observation of two classroom sessions at Grade 3 (age 8 years) on multiplicative situations (planned by the didacticians and one of the teachers)
- a post-session meeting (reflection) between the sessions

Target knowledge:

Understanding situations with equal-sized groups put together in terms of multiplication, and being able to write the result as a product

→ E.g. 5·3 understood as "five threes", or "five groups of three", or "five groups with three (objects) in each group.

Teacher's goal: the students should **"**write an arithmetic representation that fits with the task".

Tasks

Task 1

Class 3c plan to arrange a class party in the Café. The day before the party, they will bake muffins for the party at school. Ms. Hall has to go the grocery store to buy eggs for the muffins. The recipe says there should be four eggs in one portion. The students have decided that they will bake twelve portions of muffins. How many eggs should Ms. Hall buy?

Task 2

The muffins are placed on baking trays to be baked in the oven. On a baking tray there is space for five rows of muffins, and there is space for seven muffins in each row. How many muffins can be placed on one baking tray?

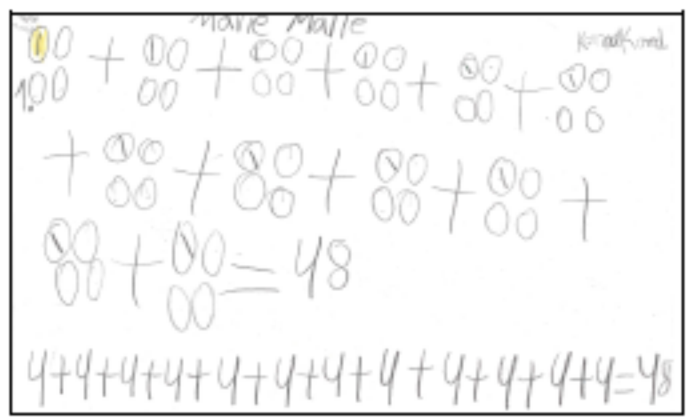
Session 1 - Implementation of Task 1

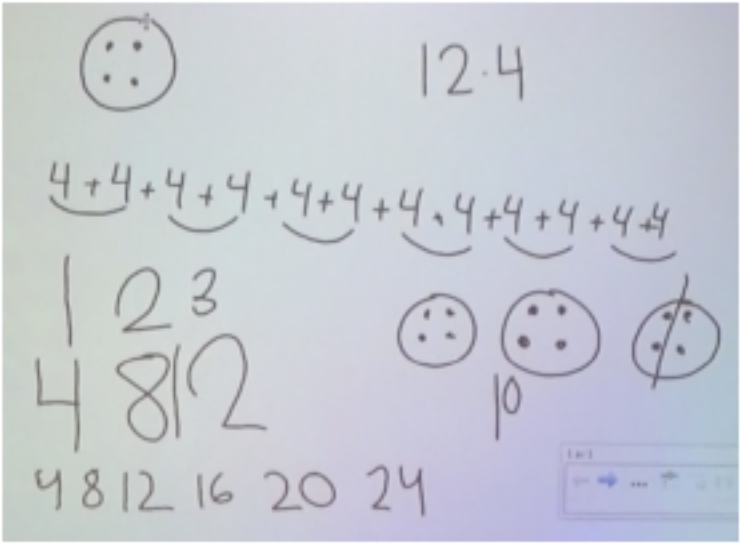
Implementation of Task 1 was divided into three phases (Video-recorded):

(1) the students' iconic representations of the situation

(2) the students' arithmetic representations of the situation

(3) the teacher's introduction of the conventional notation (12·4)





Smart Board capture with different representations

Post-session meeting right after Session 1 (audio-recorded):

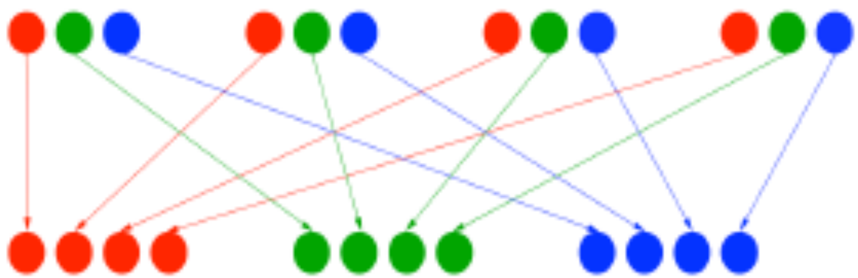
The teacher: "It was challenging to sum up, at the end of the lesson, what multiplication actually *means* – the order of numbers in a product, and what the numbers mean."

→ discussion of the order of factors, where the teacher said that it is not possible to switch the order of the factors (in Task 1) without losing the meaning of the situation.

Reflection between sessions 1 and 2

Reinterpretation of the situation by the team:

4·12 might be interpreted as 4 groups of 12, where the first group consists of the first egg with 12 portions, the second group consists of the second egg with 12 portions, and likewise for the third and fourth groups.



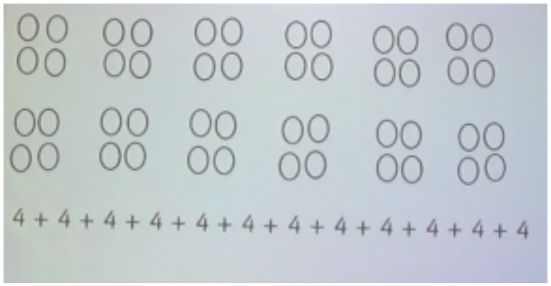
Symmetrisation of an asymmetric situation: $4 \cdot 3 \rightarrow 3 \cdot 4$

By considering such a situation, the teacher came to see that:

- Task 1 could be used to establish the *convention* regarding the order of the factors.
- Task 2 would then be used to establish *commutativity*.

SESSION 2 – THE SHIFT TO PRODUCT NOTATION

- The teacher's goal was that the students should learn to write 4+4+4+4+4+4+4+4+4+4+4+4 as 12 ·4 (not 4 ·12).



Teacher:	Why did this arithmetic problem [points at the sum of 4s] fit with what we were doing when we were making twelve portions with four eggs in each portion? Nora?
Nora:	Because it should be four eggs twelve times.

- Discussion: "twelve times four" or "four twelve times" or "twelve four times"?
- Led the teacher to write:

4+4+4+4+4+4+4+4+4+4+4+4	12+12+12+12
12 ·4	4 ·12

$12 \cdot 4$ VERSUS $4 \cdot 12$

Teacher: … What do you think is the difference, Tanya?

Tanya: If four is written first, then you shall take twelve four times, and if twelve is written first, then you shall take erm… four twelve times.

Teacher: Aha. So, the first number [points at the first factor in the product $12 \cdot 4$]… what does the first number in the multiplication problem tell us? [Pause 6 sec.]. These numbers do not tell us the same. What is does the first number in the arithmetic problem tell us? [Pause 3 sec.] Lucas?

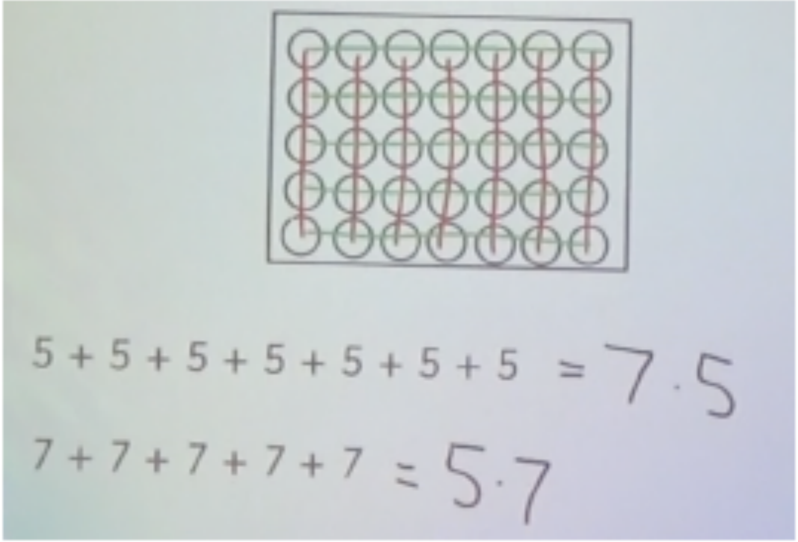
Lucas: How many we shall take.

Teacher: How many we shall take is what the first number tells us. What does the second number in the arithmetic problem tell us then? [Pause 6 sec.]. Nora?

Nora: It is the number that we shall take as many times the first number.

ARGUING FOR PRODUCT NOTATION

- Teacher suggests thinking of a thousand portions of muffins (each with 4 eggs)
- Students replied "a thousand fours"
- But one student argued: "Now you take a thousand four times. 1000+1000+1000+1000.
  - → Commutativity issue
- The teacher ended the Smart Board session with a review of Task 2



Task 2
The muffins are placed on
baking trays to be baked in the oven.
On a baking tray there is space for
five rows of muffins, and there is space
for **seven muffins in each row**. How many
muffins can be placed on one baking tray?

CONCLUDING COMMENTS

- Analysis reveals tensions between multiplicative constructs, conventional notation and situational models
- The teacher wanted the students to learn a convention of multiplication, related to situations of equivalent groups → a non-commutative situation
- However, with 1000 portions with 4 eggs in each, it is easier to calculate the total of eggs as 1000+1000+1000+1000 → creates a conflict with the desired convention
- Important for the team's learning: literature (Grouwes, 1992) in preparation for teaching sessions → teacher in post-session meeting

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Thank you for your attention!

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