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Interpreting combinatorial problems as multiplicative structures

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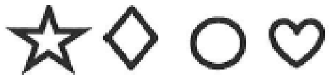
Language Use and Development in the Mathematics Classroom

- Study pupils’ mathematical language to gain knowledge that will help the teachers to develop their teaching
- Increase pupils’ proficiency in expressing mathematical ideas, reasoning, arguing and justification
- Collaboration with two primary schools
- In this presentation: 9 year old pupils (grade 4)
- Based on children’s work with two combinatorial tasks

The tasks

How many different gingerbread biscuits can we make if we have cutters in these four shapes



and we have white, green and red icing?

Task 1

Ms. Hall has 3 pairs of trousers and 5 sweaters. The trousers are in the colours blue, black, and grey. The sweaters are in the colours blue, red, black, green and purple. She will use one pair of trousers and one sweater each day, and she will combine different pairs of trousers with different sweaters. How many days in a row can Ms. Hall wear different outfits?

Task 2

Research questions

- What kind of semiotic representations do children invent to describe the given problem situations?
  - How do they differ?
- To what extent and it what ways do they see the situations as multiplicative?

Multiplicative structures

- For a situation to be established as multiplicative, it is necessary to at least coordinate two composite units in such a way that one of the composite units is distributed over the elements of the other composite unit. (Steffe, 1994).

Multiplicative structures

- Isomorphy of measures
- Product of measures
- Multiple proportions
  (Vergnaud, 1983)

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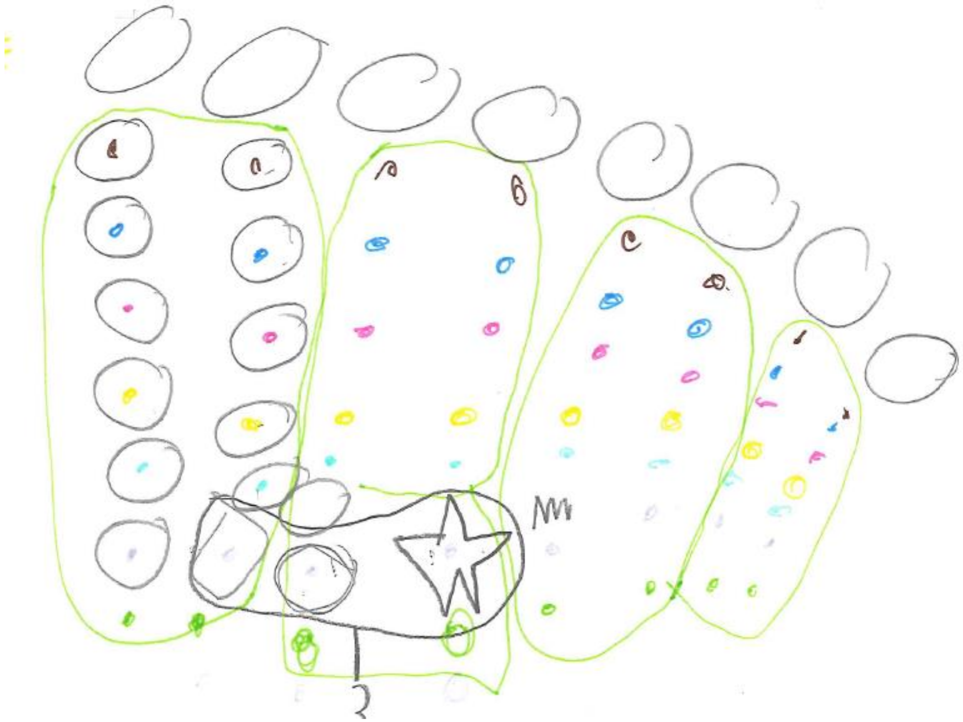
- Product of measures includes problems concerning area, volume, Cartesian product, work, and many other physical concepts. (Vergnaud,1983)

Combinatorial problems

- Multiplicative structures with a counting unit that is not present from the beginning
- The counting unit is of indefinite quantity – not clear from the beginning when to stop counting

Solution to the (extended) biscuit problem



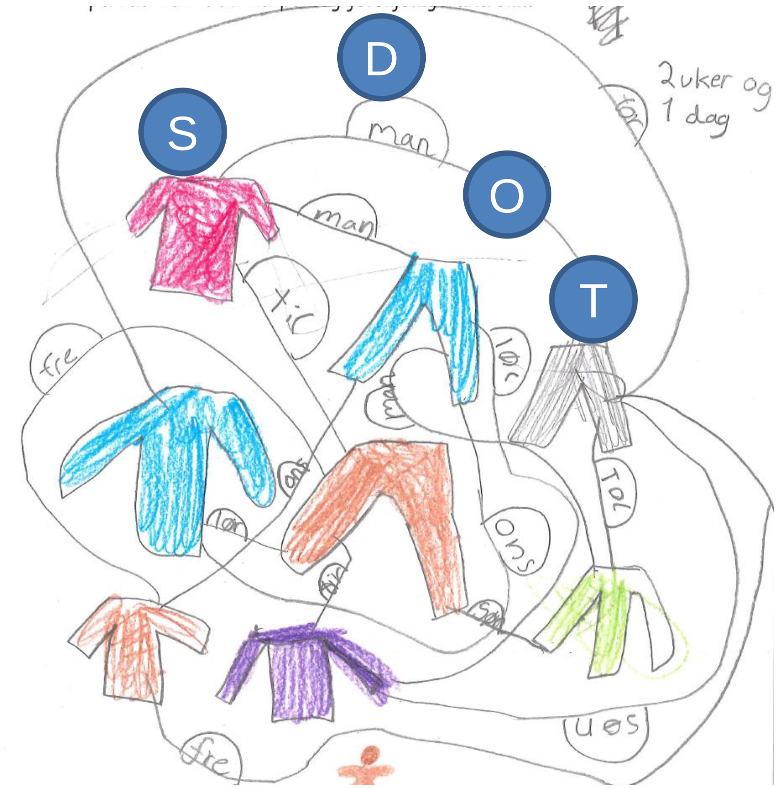
7 x 8

$$M_1 \times M_2 \rightarrow M_3$$

The countable unit is coloured biscuits

$$M_3 \; @ \; M_2$$

Solution to the outfit problem



$$M_1 \times M_2 \rightarrow M_3 \rightarrow M_4$$

Sweaters Trousers Outfits Days

Expressing the multiplicative structure

- Roger: We have been thinking that we can take all the sweaters with all the trousers and all the trousers with all the sweaters.
- Researcher: Have you thought about how many it will be?
- Roger: Fifteen. Five sweaters three times, that is fifteen.
- Researcher: So you think it is 'times'.
- Roger: Yes, because five times three is fifteen. And there are five sweaters and all five sweaters can be used three times. So then it is five times three.

References

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Collaborative tool-mediated talk – an example from third graders

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Language use and development in the mathematics classroom

Intervention project together with two primary schools where we explore and develop teaching and learning in mathematics.

Study pupils' development of mathematical proficiency, with particular focus on the role of language.

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Research question for this paper

What stimulates mathematical progress in the collaborative process of solving a task?

Theoretical framework

- Sociocultural theory (Vygotskij, 1987)
  - The general genetic law of cultural development
  - Higher mental functioning and human actions in general are mediated by tools and signs
  - Thought and language
- Three types of talk (Barnes & Todd, 1977, Littleton & Mercer, 2010)
  - Exploratory, cumulative, disputational
- Semiotic representations (Duval, 2006)
  - Transformations (treatments, conversions)

Methodology

The empirical data for this paper is a video-recorded and transcribed 7 minutes' dialog between two girls, working on the task:

The 3rd grade will have a party at school. The day before the party, they are baking muffins. Ann is going to the store to buy eggs for the muffins. In the recipe it says that they need four eggs for one portion. The children have decided that they are going to bake twelve portions of muffins. How many eggs does Ann need to buy?

Analysis in three steps

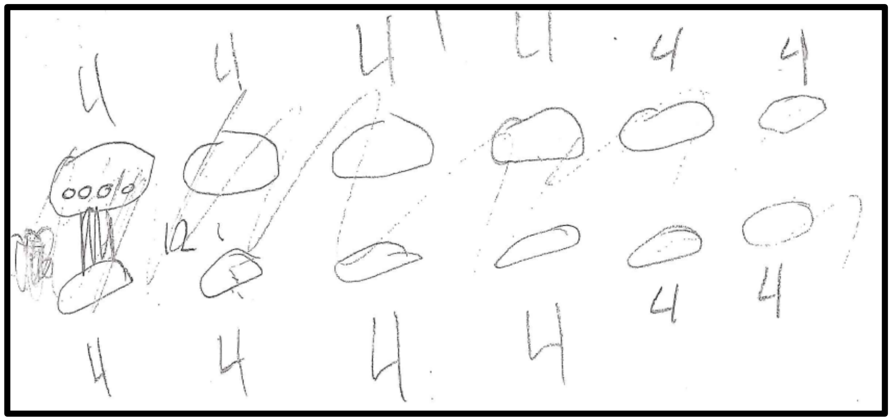
1. A conversation analysis – informed by Littleton and Mercer's (2010).

2. Identifying shifts of focus.

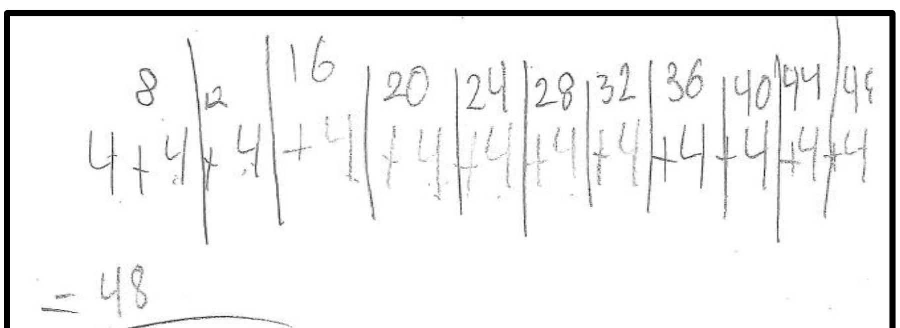
3. Analysing each sequence more thoroughly with respect to communication and mathematical progress.

What stimulates mathematical progress in the collaborative process of solving the task?

Identifying the multiplicative structure:



Calculating:



What stimulates mathematical progress in the collaborative process of solving the task?

1. Common goal in solving the task
   - Shared responsibility (repeated use of "*we*")
   - Atmosphere of trust and acknowledgement

76	B:	"So, we have to buy 48. Yes, we did it!"

44	K:	No, just... I'll… (takes the pencil from Lin). Eight plus four, we do it like this, four, four, four (writes the number 4 above each muffin).
45	B:	Can I do the last ones?
46	K:	Yes, you can do these four.
47	B:	Oh no (draws a sloppy looking 4).
48	K:	That's fine, that's fine, we can see it anyway.

What stimulates mathematical progress in the collaborative process of solving the task?

2. The girl's ablity to think out loud

3. Involvement in each others reasoning

22	K:	(Points at the four eggs) So that means four…, we should get to… we are going to have twelve. (Takes the paper from Lin.) If I draw twelve.
(…)		
25	B:	Just do it there (points right beneath the four eggs).
26	K:	I'll draw twelve muffins (starts to draw bigger circles, stops to count).
27	B:	That's funny looking muffins.
28	K:	I know, but we can see, we can see what it is anyway (completes the drawing of twelve muffins; two rows with six circles in each row).
29	B:	Now you have twelve.
30	K:	Here we have twelve muffins, and then there should be four in each muffin (points at the eggs Lin has drawn at the top of the paper).
31	B:	(Points at the four eggs) Then we put these down here, these four in one, then we have to… (points from the four eggs to the twelve muffins).

A collaborative tool-mediated talk

As the two girls are using language effectively for joint, explicit, collaborative reasoning we claim that the conversation inherits features of exploratory talk (Littleton & Mercer, 2010).

Our study adds to the field throwing light on how drawings are necessary mediational means in young learner's exploratory talk.

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